Progress in Mathematics

Logic Model

Study Type: ESSA Evidence Level IV

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EXECUTIVE SUMMARY

William H. Sadlier engaged LearnPlatform by Instructure, a third-party edtech research company, to develop a logic model for its *Progress in Mathematics* program. LearnPlatform designed the logic model to satisfy Level IV requirements (*Demonstrates a Rationale*) according to the Every Student Succeeds Act (ESSA).[1]

Logic Model

A logic model provides a program roadmap, detailing program inputs, participants reached, program activities, outputs, and outcomes. LearnPlatform collaborated with Sadlier to develop and revise the logic model for *Progress in Mathematics*.

Study Design for *Progress in Mathematics* Evaluation

Informed by the logic model, the next phase will focus on planning for an ESSA Level III study to examine the extent to which *Progress in Mathematics* impacts students' evidence-based writing.

Conclusions

This study satisfies ESSA evidence requirements for Level IV (*Demonstrates a Rationale*). Specifically, this study met the following criteria for Level IV:

- ✅ Detailed logic model informed by previous, high-quality research
- ✅ Study planning and design is currently underway for an ESSA Level III study

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[1] Level IV indicates that an intervention should include a "well-specified logic model that is informed by research or an evaluation that suggests how the intervention is likely to improve relevant outcomes; and an effort to study the effects of the intervention, that will happen as part of the intervention or is underway elsewhere…" (p. 9, U.S. Department of Education, 2016).

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Introduction

William H. Sadlier engaged LearnPlatform by Instructure, a third-party edtech research company, to develop a logic model for its *Progress in Mathematics* program. LearnPlatform revised the logic model to satisfy Level IV requirements (*Demonstrates a Rationale*) according to the Every Student Succeeds Act (ESSA).

Sadlier recognizes that proficiency in math skills and knowledge is key for long-term success in math and post-schooling outcomes. Students who struggle to acquire math proficiency in grades K–5 continue to struggle in later years losing pace with their grade-level peers. Research-based math interventions are necessary to support the acquisition of math skills and knowledge, however teachers often lack adequate training and targeted resources to meet the learning needs of each student. Progress in Mathematics provides rigorous math content focused on building deep conceptual understanding and procedural fluency equally. With explicit in-depth instruction in fundamental mathematical concepts, the program emphasizes the development of higher order thinking skills, precision in computational fluency and math vocabulary, and is supported by an abundance of practice, thus helping develop mathematically proficient students.

Explicit and Systematic Instruction. Explicit and systematic instruction refers to instructional components that are purposefully structured to gradually enhance students' knowledge, aiming for specific learning goals. Systematic intervention materials are crafted to introduce topics progressively and deliberately, with the provided instruction aiding student learning. This method is particularly beneficial for students who face learning challenges. Research on systematic instruction ensures that students build on previously learned concepts and skills, allowing for the gradual acquisition of more complex material (Fuchs et al., 2021; Steedly et al. 2008). Direct and systematic instruction depends on well-organized and structured lesson plans that unite objectives, mental mathematics, developmental activities, and homework in a logical and coherent way that makes each lesson consistent, integrated and complete (Panasuk, Stone, & Todd, 2002). As such, *Progress in Mathematics* contains a program scope and sequence that details the systematic growth of the content from one grade to the next; a table of contents that shows the sequencing of mathematics content for each grade; clear models that explicitly teach new content; and a 5-step[2] lesson plan (in the teacher edition) that supports teachers in delivering direct instruction of mathematical concepts and skills.

Conceptual Understanding. Conceptual understanding in mathematics involves helping students grasp the underlying concepts and relationships of computational procedures. This understanding is developed through a three-stage process: concrete, visual, and symbolic. Research indicates that conceptual knowledge is crucial for computational skills. Additionally, studies show that conceptual knowledge is positively correlated with computational skill (Fries et al., 2021). The use of manipulatives can also be beneficial, particularly for younger students and those with learning disabilities. Within *Progress Mathematics,* concrete presentations serve as the

[2] Lesson readiness, Teaching the lesson, Practice and apply, Summarize/assess, and Follow-up

initial phase in a three-stage process aimed at establishing a solid procedural foundation and assisting children in transitioning from concrete to visual thinking and eventually to symbolic representation. The second and third stages involve visual and symbolic presentations, respectively.

Fluency in Numerical Operations. Fluency in numerical operations is achieved through the development of number sense and extensive practice. Automaticity in performing computations accurately and efficiently frees mental energy for problem-solving. Additionally, research in reading highlights the importance of fluency for comprehension, which parallels the need for computational fluency in mathematics. Automaticity in basic arithmetic operations is essential for higher-level mathematical learning (Baroody et al. 2009; 2012; Dyson et al. 2015; 2019; Geary et al., 2007; 2008; 2012; Ketterlin-Geller at al., 2008). Sequential algorithms presented in the *Progress in Mathematics* curriculum enable children to efficiently and accurately carry out mathematical operations without the need for calculators. Additionally, extensive practice of strategies and facts is provided to develop fluency.

Problem Solving. Problem-solving in mathematics involves applying reading skills, conceptual knowledge, and computational skills to solve non-routine problems. Research emphasizes the importance of spatial representation, reading skills, and teacher guidance in problem-solving. Solving word problems is a crucial aspect of the elementary mathematics curriculum as it enables students to apply their mathematical knowledge, enhance critical thinking skills, and relate mathematics to different situations. Mastering word problems can deepen students' comprehension of grade-level content and prepare them for success in higher-level math courses and future careers (Fuchs et al . 2019; Jitendra et al. 2013; Powell et al. , 2015). Developing strategies step-by-step, with each phase of the problem-solving process is clearly demonstrated in *Progress in Mathematics.* Thus, providing opportunities for analysis and review, where children examine information and use previously learned strategies to solve problems involving various types of numbers.

Critical Foundations in Algebra. Scholars have long emphasized the importance of noticing and reasoning with mathematical structures as essential components of algebraic thinking, though these skills are challenging for students to master (e.g., Kieran, 2007; Knuth et al., 2006; Pimm, 1995). Unfortunately, elementary arithmetic instruction often overlooks structural reasoning, missing opportunities for young learners (Arcavi, et al., 2017). Introducing algebraic thinking early, alongside concepts of space, measure, data, and chance, helps students see how numbers form systems and generalize patterns. Effective number study should focus on understanding operations conceptually and their real-world applications. Students should gain experience in representing, abstracting, and generalizing numerical relationships before formally studying algebra (National Research Council, 2001). Progress in Mathematics provides a curriculum that can be implemented to achieve mastery of all of the Benchmarks for the Critical Foundations of Algebra by the end of Grade 7, including a scope and sequence that is designed for timely mastery of the critical foundations of Algebra in accordance with the recommendations of the National Mathematics Advisory Panel pacing that identifies the foundational concepts for Algebra as the priority curriculum early introduction of the rudiments of algebra, including the notions of variable and function.

Vocabulary Development. Mathematical language is a form of academic language that expresses mathematical concepts. It includes the vocabulary, terminology, and structures used in thinking, discussing, and writing about mathematics. This language provides a more precise understanding of mathematics compared to everyday conversational language. Understanding mathematical language is essential for students because it is used in textbooks, curriculum materials, assessments, and instruction. By teaching mathematical language, teachers help students grasp subtle and complex mathematical ideas. Emphasizing mathematical language during interventions also aids students in accessing the language used in core instruction. Therefore, developing students' mathematical language is crucial for their success, especially as the material becomes more advanced. When both teachers and students use mathematical language, communication in the classroom becomes clearer. As teachers model correct mathematical language, students learn how these terms fit with the mathematics they are studying and start incorporating this language into their own explanations (Bay-Williams & Livers, 2009; Capraro & Joffrion, 2006; Clarke et al., 2017; Dunston & Tyminski, 2013; Fuchs & Fuchs, 2001; Monroe & Orme, 2002; Powell & Driver, 2015). Within *Progress in Mathematics*, **Math Words** teach children the language of mathematics and how to communicate mathematically. Talk It Over activities connect children's understanding of math concepts with math vocabulary, while **Do You Remember?** exercises review previously learned vocabulary. **Math Vocabulary** activities in the Teacher's Edition engage children in discussions using math terms, and **Write About It** activities encourage children to use math vocabulary in their writing. Additionally, an online grade-level glossary illustrates and defines program math vocabulary.

Practice and Review. Regular practice and review are essential for mastering mathematical skills and ensuring long-term retention. Practice should be guided initially, differentiated to challenge advanced students and support slower learners, and then undertaken independently. Sustained practice, or ongoing review, helps maintain fluency in skills. Research supports the importance of practice for skill retention and long-term memory. Studies emphasize that extensive practice is necessary for procedural learning (Ericsson et al. 2018). Regular practice and review activities, such as mental math exercises and cumulative reviews, are effective in sustaining fluency. The *Progress in Mathematics* program offers practice and review through: Guided practice helps children move towards independent practice; **Independent practice** opportunities enable mastery of new skills; **"Do You Remember?"** exercises offer mixed reviews of past skills; **Cumulative Review** tests cover previous chapter materials; **Lesson Readiness** activities in the Teacher's Edition review skills and maintain fluency; and **Mental Math and Problem of the Day** exercises provide daily practice.

Formative and Summative Assessments and Differentiated Instruction. Assessments play a crucial role in guiding instruction and measuring student progress ((Bransford et al., 2000). Formative assessments provide ongoing information about individual progress, helping teachers adjust instruction as needed (NMAP, 2008). Summative assessments measure student achievement in relation to external standards. Research highlights the value of frequent formative assessments and the inclusion of high-level content in summative assessments. Studies show that frequent assessments, combined with differentiated instruction or instructional adjustments, can significantly improve student performance (Black & Wiliam,1998b; Brookhart, 2015; Hattie, 2009; Heritage, 2007). Effective assessments should provide opportunities for feedback and revision, aligning with learning goals (Hattie & Timperley, 2007). Teachers can use a suite of formative and summative assessments in the *Progress in Mathematics* program to assess students. Data from these assessments and reteaching suggestions in the Teacher's Edition inform teachers about specific ways of using formative assessment information to provide differentiated instruction. This edition provides differentiated activities that teachers can use to support the specific needs of multilingual learners, students struggling with specific topics, and enrichment opportunities for those students who are ready to explore mathematical concepts in more depth.

A prior review of the literature conducted by Stotsky (2009) can be found here.

Logic Model

A logic model is a program or product roadmap. It identifies how a program aims to impact learners, translating inputs into measurable activities that lead to expected results. A logic model has five core components: inputs, participants, activities, outputs, and outcomes (see Table 1).

Table 1. Logic model core components

Component	Description	More information
Inputs	What the provider invests	What resources are invested and/or required for the learning solution to function effectively in real schools?
Participants	Who the provider reaches	Who receives the learning solution or intervention? Who are the key users?
Activities	What participants do	What do participants do with the resources identified in Inputs? What are the core/essential components of the learning solution? What is being delivered to help students/teachers achieve the program outcomes identified?
Outputs	Products of activities	What are numeric indicators of activities? (e.g., key performance indicators; allows for examining program implementation)
Outcomes	Short-term, intermediate, long-term	Short-term outcomes are changes in awareness, knowledge, skills, attitudes, and aspirations. Intermediate outcomes are changes in behaviors or actions. Long-term outcomes are ultimate impacts or changes in social, economic, civil or environmental conditions.

LearnPlatform reviewed *Progress in Mathematics* resources, artifacts, and program materials to develop a draft logic model. William H. Sadlier reviewed the draft and provided revisions during virtual meetings. The final logic model depicted below (Figure 1) reflects these conversations and revisions.

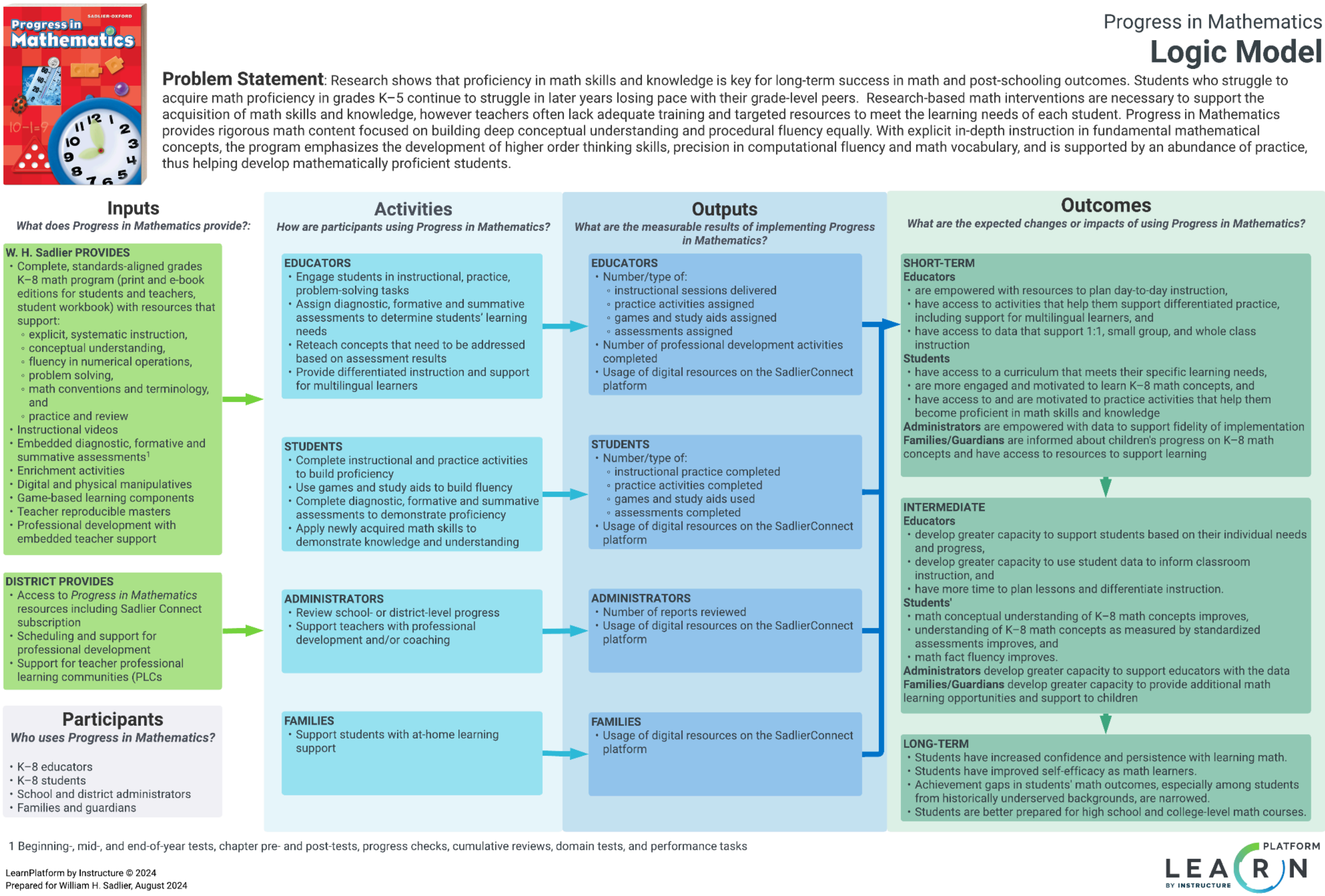

Figure 1. *Progress in Mathematics* logic model

Progress in Mathematics Logic Model Components. William H. Sadlier invests several resources into their program, including:

- Complete, standards-aligned grades K–5 math program (print and e-book editions for students and teachers, student workbook) with resources that support:
    - explicit, systematic instruction,
    - conceptual understanding,
    - fluency in numerical operations,
    - problem solving,
    - math conventions and terminology, and
    - practice and review
- Instructional videos
- Embedded diagnostic, formative and summative assessments
- Enrichment activities
- Digital and physical manipulatives
- Game-based learning components
- Teacher reproducible masters
- Professional development with embedded teacher support

Ultimately, the *Progress in Mathematics* program aims to reach K–5 educators, K–5 students, school and district administrators, and families and guardians. Using these program resources, participants can engage in the following activities:

- Educators
    - Engage students in instructional, practice, problem-solving tasks
    - Assign diagnostic, formative and summative assessments to determine students' learning needs
    - Reteach concepts that need to be addressed based on assessment results
    - Provide differentiated instruction and support for multilingual learners
- Students
    - Complete instructional and practice activities to build proficiency
    - Use games and study aids to build fluency
    - Complete diagnostic, formative and summative assessments to demonstrate proficiency
    - Apply newly acquired math skills to demonstrate knowledge and understanding
- Administrators
    - Review school- or district-level progress
    - Support teachers with professional development and/or coaching
- Families/Guardians support students with at-home learning support

William H. Sadlier can examine the extent to which core activities were delivered and participants were reached by examining the following quantifiable outputs:

- Educators
    - Number/type of:
        - instructional sessions delivered

        - practice activities assigned
        - games and study aids assigned
        - assessments assigned
    - Number of professional development activities completed
    - Usage of digital resources on the SadlierConnect platform
- Students
    - Number/type of:
        - instructional sessions attended
        - practice activities completed
        - games and study aids used
        - assessments completed
    - Usage of digital resources on the SadlierConnect platform
- Administrators
    - Number of reports reviewed
    - Usage of digital resources on the SadlierConnect platform
- Families/Guardians
    - Usage of digital resources on the SadlierConnect platform

If implementation is successful, based on a review of program outputs, Sadlier can expect the following short-term outcomes. In the shorter term, educators are empowered with resources to plan day-to-day instruction, have access to activities that help them support differentiated practice, including support for multilingual learners, and have access to data that support 1:1, small group, and whole class instruction. Additionally, students have access to a curriculum that meets their specific learning needs, are more engaged and motivated to learn K–5 math concepts, and have access to and are motivated to practice math activities that help them become proficient in math skills and knowledge. Administrators are empowered with data to support fidelity of implementation and likewise, families or guardians are informed about children's progress on K–5 math concepts and have access to resources to support learning.

In the intermediate term, Educators will be able to develop greater capacity to support students based on their individual needs and progress and to use student data to inform classroom instruction. They will also have more time to plan lessons and differentiate instruction. Students' math conceptual understanding of K–5 math concepts, understanding of K–5 math concepts as measured by standardized assessments, and math fact fluency will improve. Administrators will develop greater capacity to support educators with the data while families or guardians will develop greater capacity to provide additional math learning opportunities and support to children.

In the Long-term, students have increased confidence and persistence with learning math and have improved self-efficacy as math learners. Subsequently, achievement gaps in students' math outcomes, especially among students from historically underserved backgrounds, are narrowed. Overall, students are better prepared for high school and college-level math courses.

Study Design for *Progress in Mathematics* Evaluation

To continue building evidence of effectiveness and to examine the proposed relationships in the logic model, *Progress in Mathematics* has plans to conduct an evaluation to determine the extent to which its program produces the desired outcomes. Specifically, Sadlier has plans to begin an ESSA Level III study to answer the following research questions:

Implementation Questions

1. Among teachers, what were the usage patterns of *Progress in Mathematics*? On average, how many lesson plans were completed?
2. To what extent did teachers implement the 5-step Lesson Plan?

Outcome Questions

3. After controlling for students' prior math achievement, how was teachers' use of *Progress in Mathematics* related to:
    a. students' mastery of math standards within the *Progress in Mathematics* program?
    b. students' mastery of assigned lessons?
4. After controlling for students' prior math achievement, how was teachers' use of *Progress in Mathematics* related to students' performance on standardized math assessments?

Conclusions

This study satisfies ESSA evidence requirements for Level IV (*Demonstrates a Rationale*). Specifically, this study met the following criteria for Level IV:

✅ Detailed logic model informed by previous, high-quality research
✅ Study planning and design is currently underway for an ESSA Level I, II or III study

References

Baroody, A. J., Bajwa, N. P., & Eiland, M. (2009). Why can't Johnny remember the basic facts? *Developmental Disabilities Research Reviews, 15*(1), 69–79. https://eric.ed.gov/?id=EJ859019

Baroody, A. J., Eiland, M. D., Purpura, D. J., & Reid, E. E. (2012). Fostering at-risk kindergarten children's number sense. *Cognition and Instruction, 30*(4), 435–470. https://eric.ed.gov/?id=EJ982662

Bay-Williams, J. M., & Livers, S. (2009). Supporting math vocabulary acquisition. *Teaching Children Mathematics, 16*(4), 238–245. https://eric.ed.gov/?id=EJ862806

Black, P. J., & Wiliam, D. (1998). Inside the Black Box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*, 139-48.

Bransford, J.D., Brown, A.L., & Cocking, R.R. (Eds.). (2000). *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: The National Academy Press, 139–140.

Brookhart, S. M. (2015). *How to make decisions with different kinds of student assessment data.* Alexandria, VA: ASCD.

Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology, 27*(2–3), 147–164. https://eric. ed.gov/?id=EJ737448

Clarke, B., Doabler, C. T., Kosty, D., Kurtz-Nelson, E., Smolkowski, K., Fien, H., & Turtura, J. (2017). Testing the efficacy of a kindergarten mathematics intervention by small group size. *AERA Open, 3*(2), 1–16. https://eric.ed.gov/?id=EJ1194151

Dunston, P. J., & Tyminski, A. M. (2013). What's the big deal about vocabulary? *Mathematics Teaching in the Middle school, 19*(1), 38–45. https://eric.ed.gov/?id=EJ1026045

Dyson, N., Jordan, N. C., Beliakoff, A., & Hassinger-Das, B. (2015). A kindergarten number sense intervention with contrasting practice conditions for low-achieving children. *Journal for Research in Mathematics Education, 46*(3), 331–370. https://eric.ed.gov/?id=EJ1088271

Dyson, N. I., Jordan, N. C., Rodrigues, J., Barbieri, C., & Rinne, L. (2018). A fraction sense intervention for sixth graders with or at risk for mathematics difficulties. *Remedial and Special Education.* Advance online publication. https://eric.ed.gov/?id=ED603994

Ericsson, K. A., Hoffman, R. R., & Kozbelt, A. (Eds.). (2018). *The Cambridge handbook of expertise and expert performance*. Cambridge University Press.

Fries, L., Son, J. Y., Givvin, K. B., & Stigler, J. W. (2021). Practicing connections: A framework to guide instructional design for developing understanding in complex domains. *Educational Psychology Review, 33*(2), 739-762.

Fuchs, L. S., Bucka, N., Clarke, B., Dougherty, B., Jordan, N. C., Karp, K. S., ... & Morgan, S. (2021). Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades. Educator's Practice Guide. WWC 2021006. *What Works Clearinghouse.*

Fuchs, L. S., & Fuchs, D. (2001). Principles for the prevention and intervention of mathematics difficulties. *Learning Disabilities Research & Practice, 16*(2), 85–95. https://eric.ed.gov/?id=EJ630961

Fuchs, L. S., Seethaler, P. M., Sterba, S. K., Craddock, C., Fuchs, D., Compton, D. L., Geary, D. C., & Changas, P. (2019). *Schema-based word-problem intervention with and without embedded language comprehension instruction.* Vanderbilt University

Geary, D. C., Boykin, A. W., Embretson, S., Reyna, V., Siegler, R., Berch, D. B., & Graban, J. (2008). Chapter 4: Report of the task group on learning processes. U.S. Department of Education. https://www2.ed.gov/about/bdscomm/list/mathpanel/reports.html

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. Child Development, 78(4), 1343–1359. https://eric.ed.gov/?id=EJ769869

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. Journal of Educational Psychology, 104(1), 206–223. https://doi.org/10.1037/a0025398

Hattie, J. A. (2009). *Visible learning: A synthesis of 800+ meta-analyses on achievement.* New York, NY: Routledge.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81–112.

Heritage, M. (2007). Formative assessment: What do teachers need to know and do?. *Phi Delta Kappan, 89*(2), 140-145.

Jitendra, A. K., Dupuis, D. N., Rodriguez, M. C., Zaslofsky, A. F., Slater, S., Cozine-Corroy, K., & Church, C. (2013). A randomized controlled trial of the impact of schema-based instruction on mathematical outcomes for third-grade students with mathematics difficulties. *Elementary School Journal, 114*(2), 252–276. https://eric.ed.gov/?id=EJ1015539

Ketterlin-Geller, L. R., Chard, D. J., & Fien, H. (2008). Making connections in mathematics: Conceptual mathematics intervention for low-performing students. *Remedial and Special Education, 29*(1), 33–45. https://eric.ed.gov/?id=EJ796940

Kieran, C. (2007). Learning and teaching of algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 707–762). Charlotte, NC: Information Age.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*, 297–312.

Monroe, E. E., & Orme, M. P. (2002). Developing mathematical vocabulary. *Preventing School Failure: Alternative Education for Children and Youth, 46*(3), 139–142. https://eric.ed.gov/?id=EJ648803

National Mathematics Advisory Panel. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel,* U.S. Department of Education: Washington, DC, 2008.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Panasuk, R., Stone, W., & Todd, J. (2002). Lesson Planning Strategy for Effective Mathematics Teaching. *Education, 122*(4), p 809.

Pimm, D. (1995). *Symbols and meanings in school mathematics*. London, England: Routledge. doi:10.4324/9780203428610

Powell, S. R., Driver, M. K., & Julian, T. E. (2015). The effect of tutoring with nonstandard equations for students with mathematics difficulty. *Journal of Learning Disabilities, 48*(5), 523–534. https://eric.ed.gov/?id=EJ1070884

Powell, S. R., Fuchs, L. S., Cirino, P. T., Fuchs, D., Compton, D. L., & Changas, P. C. (2015). Effects of a multitier support system on calculation, word problem, and prealgebraic performance among at-risk learners. *Exceptional Children, 81*(4), 443–470. https://eric.ed.gov/?id=EJ1065063

Steedly, K., Dragoo, K., Arafeh, S., & Luke, S. D. (2008). Effective mathematics instruction. *Evidence for Education, 3*(1), 2–11. National Dissemination Center for Children with Disabilities. https://files.eric.ed.gov/fulltext/ED572704.pdf