Progress in Mathematics™

RESEARCH BASE

by

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Contents

Support for *Progress in Mathematics*: Highlights from Research Studies

Signed into law in January 2002, the *No Child Left Behind Act* contains the most comprehensive reforms of the *Elementary and Secondary Education Act* since its enactment in 1965. The *Act* emphasizes the use of research-supported teaching methods, learning activities, and curriculum materials in all disciplines, but especially in reading and mathematics, the two basic subjects in the school curriculum. Indeed, the *Act* requires that federal funding go only to those programs that are supported by evidence from scientific research studies.

Teachers can use three different types of evidence from scientifically based research to make curricular and instructional decisions. The authors of a monograph for the National Institute for Literacy (Stanovich & Stanovich, 2003) note that "evidence of instructional effectiveness can come from any of the following sources:

- Demonstrated student achievement in formal testing situations implemented by the teacher, school district, or state;
- Published findings of research-based evidence that the instructional methods being used by teachers lead to student achievement; or
- Proof of reason-based practice that converges with a research-based consensus in the scientific literature. This type of justification of educational practice becomes important when direct evidence may be lacking…, but there is a theoretical link to research-based evidence that can be traced."

The purpose of this document is to show how the major instructional and programmatic features of *Progress in Mathematics* are supported by scientifically sound research reviews and individual studies. The document does so in two ways. First, it identifies the research base in mathematics education that supports the generic instructional and programmatic features of *Progress in Mathematics*. Second, it identifies the research base in reading education that further supports these generic instructional and programmatic features. The many parallels in teaching strategies and learning processes between beginning mathematics and beginning reading are one reason for identifying this research base in reading. It will be useful to teachers who teach both mathematics and reading in self-contained classrooms from kindergarten through grade 6 to learn what these parallels are and how research findings in elementary reading can supplement research findings in elementary mathematics. A second reason for identifying the supportive research base in reading instruction is the status of the body of research in mathematics education. It is much smaller and newer than the body of research on reading and, by itself, does not provide sufficient evidence to guide policy or practice. Moreover, after a comprehensive review of the studies evaluating the 13 mathematics curricula funded by the National Science Foundation and six publisher-generated programs produced during the 1990s, the reviewing committee concluded that no valid body of studies supports the effectiveness of any of those programs (National Research Council, 2004).

The *Progress in Mathematics* series was developed in the 1940s by the Sisters, Servants of the Immaculate Heart of Mary who taught in the Catholic schools of Philadelphia, Harrisburg, and Scranton. As a result, the series is used in many Catholic elementary schools across the country. It is also used in many more Catholic elementary schools than its nearest competitor.

It is informative to set Catholic schools in the context of the national profile of student achievement in mathematics in the elementary grades. In 2003, results from the Grade 4 Mathematics Assessment in the *National Assessment of Educational Progress (NAEP)* were broken down by type of school, a category that was divided into seven kinds of schools. As Table 1 shows, grade 4 students in Catholic schools scored significantly higher than grade 4 students in public schools, achieving an average scale score of 244 in contrast to an average scale score of 234 for the public school students. Table 1 also shows that a significantly higher percentage of the students in Catholic schools scored at or above the Proficient level compared to students in public schools, 43% to 31%. Thus, *Progress in Mathematics* is used in a category of schools whose students overall score higher at all performance levels than do those in public schools.

Progress in Mathematics has been published for more than 60 years by William H. Sadlier, Inc. The original authors of the program proposed a number of principles to guide the pedagogy used for teaching mathematics in the elementary school. Their far-sighted vision of what a sound elementary mathematics program should contain continues to guide *Progress in Mathematics.* Their guiding principles can be articulated using the letters in the program's name and are explained in detail here.

P = Powerful development of problem-solving skills and strategies

R = Reality-based on what works in the classroom because it is developed by teachers for teachers

O = Ongoing review and maintenance of prior knowledge of concepts, skills, and problem solving

G = Grounded in a dedication to lead students from concrete understanding to pictorial representation and finally to abstract thought

R = Reaching students wherever they are in their cognitive and mathematical development, with specific suggestions for reteaching some, reinforcing learning in all, and enriching and challenging others

E = Evaluation tools that assess conceptual understanding, mathematical vocabulary, skill development, and problem solving through varied methods

S = Sequential development of mathematical concepts, skills, processes, problem solving, and reasoning

S = Substantial practice of newly acquired concepts and skills through abundant and varied exercises, problems, and mental math, with strong reliance on students' explanation of thinking orally and in writing

Table 1: National/Mathematics Composite/Grade 4/2003

Type of school as categorized by one of seven types (2002 and later): Public, other private, Catholic, Bureau of Indian Affairs, Department of Defense Education Activity, Lutheran, Conservative Christian

Percentage of Students At or Above Each Achievement Level
(with Standard Errors in Parentheses)

	N	Average Scale Score	Below Basic	At or Above Basic	At or Above Proficient	At Advanced
Public	184,325	234 (0.2)	24% (0.3)	76% (0.3)	31% (0.3)	4% (0.1)
Other private	——	---- (---)	---- (---)	---- (---)	---- (---)	---- (---)
Catholic	2,285	244 (0.8)	12% (0.9)	88% (0.9)	43% (1.2)	5% (0.7)
Bureau of Indian Affairs	——	---- (---)	---- (---)	---- (---)	---- (---)	---- (---)
Department of Defense	1,088	237 (0.8)	16% (1.3)	84% (1.3)	32% (1.6)	2% (0.7)
Lutheran	555	245 (1.5)	10% (1.7)	90% (1.7)	46% (3.5)	5% (1.1)
Conservative Christian	——	---- (---)	---- (---)	---- (---)	---- (---)	---- (---)

—— Sample size is insufficient to permit a reliable estimate.

NOTE: The NAEP Mathematics scale ranges from 0 to 500. Observed differences are not necessarily statistically significant.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, *National Assessment of Educational Progress (NAEP)*: 2003 Mathematics Assessments.

Instructional and Programmatic Features of *Progress in Mathematics*

1. Explicit and Systematic Instruction

Explicit and systematic instruction refers to the direct teaching of mathematical concepts and skills in a clearly defined sequence designed to grow over the course of the school year and from grade to grade. A logically developed progression of concepts and skills ensures that children are able to learn more and more complex material as they build on the foundation of previously learned concepts and skills. Direct teaching of a logical sequence of concepts and skills extends children's knowledge systematically in mathematically effective ways. Nothing is left out or left to chance or random discovery. The benefits of explicit and systematic instruction are supported by research in mathematics, reading, and science education.

How research supports it...

Research Reviews: In a review of high-quality studies in mathematics education, the National Center to Improve the Tools of Educators found support for direct teaching and "guided discovery." It also found support for selecting and sequencing instructional examples according to principles of concept acquisition in studies of "effective strategies." It found no advantages for "strictly discovery instruction" (Dixon et al, 1998).

The reviewers also discerned a pattern for effective mathematics lessons in many of these studies and provided a "general model for effective lessons":

> In Phase I, teachers demonstrate, explain, question, and/or conduct discussions. Students are actively involved, through answering questions and/or discussion.
>
> In Phase II, teachers, individual peers, and/or groups of peers provide students with substantial help that is gradually reduced. Students receive feedback on their performance, correctives, additional explanations, and other forms of assistance.
>
> In Phase III, teachers assess students' ability to apply knowledge to taught and/or untaught problems. Students demonstrate their ability to independently recall and/or generalize and transfer their knowledge.

In his own review of high-quality research in mathematics education, the Director of the Institute of Education Sciences found that "direct instruction can help students learn computational skills and understand math principles."

> We know that children don't have to discover math principles on their own…in order to understand mathematical concepts (Whitehurst, 2003).

The review of research on beginning reading instruction, in a monograph distributed by the U.S. Department of Education as part of the Reading First initiative (Armbruster & Osborn, 2001), also offers strong support for explicit instruction. It notes four steps that are basic to explicit instruction. These four steps, which are essential for successful learning experiences, can take place in whatever order is appropriate to the topic and cognitive level of the child.

> **Direct Instruction:** The teacher helps children become aware of what they are to learn and how they are to use their new knowledge.
>
> **Modeling:** The teacher demonstrates or models the thinking process, using a think-aloud technique.
>
> **Guided Practice:** The teacher guides the children and assists them as they learn how and when to apply their new knowledge.
>
> **Application:** The teacher helps children apply their new knowledge in extended activities.

Individual Studies: Direct instruction may work and generalize as well in science education as in reading and mathematics education. In a study of direct instruction versus exploration in science learning, the researchers found that third and fourth graders whose teachers controlled the goals, materials, examples, explanations, and pace of instruction could design unconfounded experiments and critically evaluate flawed experiments better than third and fourth graders whose teachers did no more than suggest a learning objective.

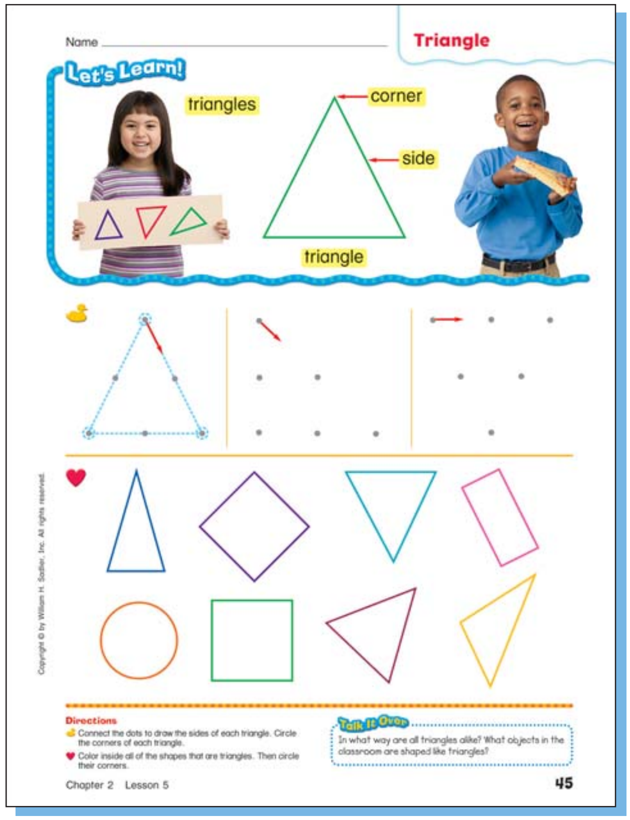
We found not only that many more children learned from direct instruction than from discovery learning, but also that when asked to make broader, richer scientific judgments, the many children who learned about experimental design from direct instruction performed as well as those few children who discovered the method on their own. These results challenge predictions derived from the presumed superiority of discovery approaches in teaching young children basic procedures for early scientific investigations (Klahr & Nigam, 2004).

Direct and systematic instruction depends on good lesson planning. Well-organized lessons also affect how new knowledge is constructed.

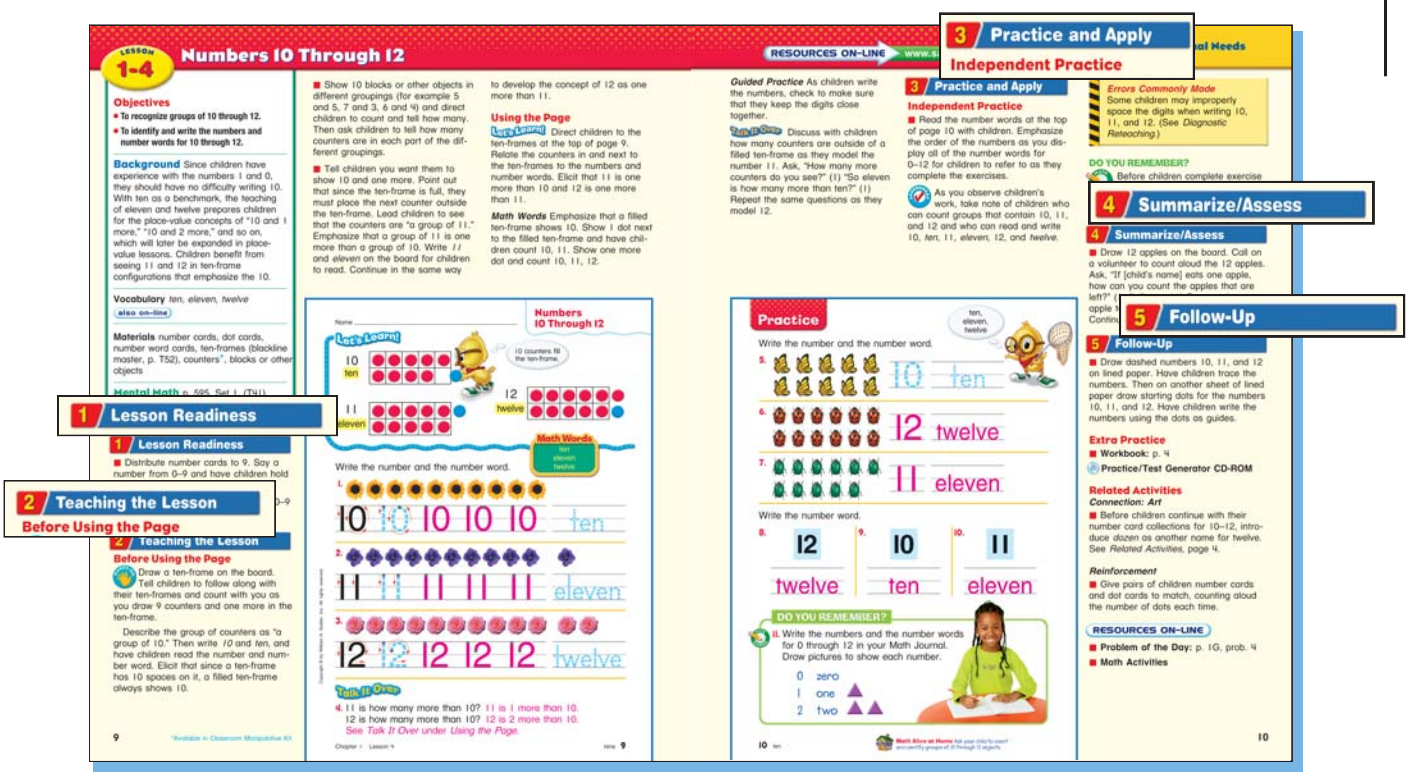
Carefully structured lesson plans specify such core components as objectives, mental mathematics, developmental activities, and homework. All the core components are basic to planning. The lesson plan is a design uniting them in a logical and coherent way that makes each lesson consistent, integrated and complete (Panasuk, Stone, & Todd, 2002).

Progress in Mathematics responds with...

- A **Program Scope and Sequence** that details the systematic growth of the content from grade level to grade level (available online)
- A **Table of Contents** that shows the careful sequencing of mathematics content for each grade (not shown)
- **Clear models** that explicitly teach new content
- A **5-Step Lesson Plan** (in the Teacher's Edition) that delivers direct instruction of mathematical concepts and skills



Grade K Student Edition page 45 **(Clear Models)**



Grade 1 Teacher's Edition pages 9–10 **(5-Step Lesson Plan)**

2. Conceptual Understanding

One major goal of mathematical learning in the elementary school is to have children understand and master the concepts and relationships underlying basic computational procedures. Children's understanding may be developed through a three-stage process that goes from concrete thinking to visual thinking and then to symbol use. In the first stage, children learn concrete meanings or referents for the central concepts and relationships in arithmetical computation through hands-on activities with a variety of manipulatives. Children relate tactile experiences designed to illustrate new concepts or relationships with what they already know or understand.

After acquiring experience-based knowledge of abstract mathematical concepts and relationships, children learn to understand visual representations of them. They interpret or construct a variety of formats for organizing numerical information such as charts, graphs, tables, or number lines. Visual representations that organize and display the significant features of abstract mathematical concepts help children make sense of them. The pictorial stage reinforces the concrete stage and prepares children for using symbols alone.

In the third stage, children manipulate the mathematical symbols that are used to communicate mathematical concepts and relationships efficiently, without the support of concrete materials or visual aids. They go beyond concrete and visual thinking to engage in more abstract thinking—the thinking on which higher levels of mathematics are based.

At each stage, children also practice basic computational procedures. Mastery in computational skills can promote as well as reflect knowledge of their underlying structures.

However, teachers need to keep in mind individual differences. The need for hands-on activities, as well as the time needed for them, depends upon the age and type of student.

Some children need many experiences with manipulatives. English language learners, especially those in the early stages of English language acquisition, will benefit from many hands-on activities, possibly with peer tutoring. Learning-disabled children may need even more time and emphasis on hands-on activities to build a firm, experience-based, intuitive understanding of the basic concepts. On the other hand, children who are not learning disabled will benefit from an early transition to symbol use to maximize the time needed for developing abstract thinking. And some students do not need hands-on activities with manipulatives at all because they can readily think abstractly; they need more challenging activities from the outset.

In the upper elementary grades, it may be helpful to group very slow learners and very fast learners separately for many of their mathematics lessons to ensure that both groups of children are appropriately challenged by their instructional materials and learning activities.

Teachers can also accommodate individual differences through scaffolded instruction. In scaffolded instruction, students are given support until they can apply new skills and strategies independently (Rosenshine & Meister, 1992). When students are first learning a new or difficult task, the teacher gives them a great deal of assistance. As they begin to demonstrate task mastery, the teacher gradually decreases assistance in order to make the students more and more responsible for their own learning.

How research supports it...

Research Reviews: A number of studies have demonstrated that conceptual understanding can be promoted through a variety of pedagogical methods and practice on a wide range of problem types. A meta-analysis of studies on research-based instructional strategies for increasing student achievement found direct instruction the most effective way to help children acquire conceptual knowledge.

> The best way to teach organizing ideas—concepts, generalizations, and principles—appears to be to present these constructs in a rather direct fashion and then have students apply these concepts, generalizations, and principles to new situations (Marzano, 1998).

A review of the relationship between conceptual and procedural knowledge found that conceptual knowledge is related to computational skill.

> Children's understanding of mathematical concepts is positively correlated with their ability to execute procedures. In some tasks, conceptual understanding precedes procedural competence; in other tasks, the order is reversed. The general lesson seems to be that the structure of the environment has a large impact on the developmental relations between conceptual and procedural knowledge (Rittle-Johnson & Siegler, 1998).

A review of the effectiveness of manipulatives in mathematics and science found that their use might improve learning in some educational contexts and for some types of students.

> Manipulatives may be a helpful tool for some student populations—especially elementary and middle school students who may have difficulty with abstract concepts in math and science content areas (for instance, students with certain learning disabilities) (Ruzic & O'Connell, 2001).

Reviews of the research on the benefits of grouping children for mathematics instruction on the basis of their level of achievement has consistently found that high-achieving students learn more when they are given more challenging content or placed in accelerated courses (Kulik, 1992; Loveless, 1998). Indeed, it is considered a matter of equity by some educators and researchers (Benbow & Stanley, 1996).

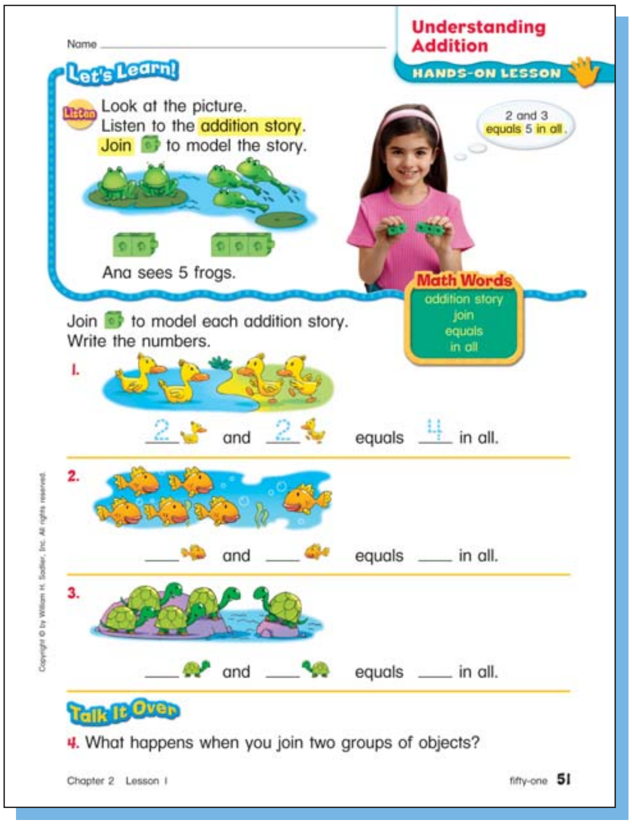
> When students are ability grouped into separate classes and given an identical curriculum, there is no appreciable effect on achievement. But when the curriculum is adjusted to correspond to ability level, it appears that student achievement is boosted, especially for high ability students receiving an accelerated curriculum. ...The elementary school practices of both within-class and cross-grade ability grouping are supported by research (Loveless, 1998).

Individual Studies: A study of the relation between conceptual knowledge and procedural knowledge noted the importance of inculcating both types of knowledge in the classroom.

> …Improved procedural knowledge can lead to improved conceptual knowledge, as well as the reverse (Rittle-Johnson, Siegler, & Alibali, 2001).

Progress in Mathematics responds with...

- **Concrete presentations** that form the first stage of a 3-stage process designed to provide a strong procedural base and help children move from concrete to visual thinking to symbol use
- **Visual presentations** and **symbolic presentations** form the second and third stages of the process



Name

Understanding Addition

HANDS-ON LESSON

Let's Learn!

Listen Look at the picture.
Listen to the addition story.
Join to model the story.

2 and 3 equals 5 in all.

Ana sees 5 frogs.

Math Words
addition story
join
equals
in all

Join to model each addition story.
Write the numbers.

1. 2 and 2 equals 4 in all.

2. ___ and ___ equals ___ in all.

3. ___ and ___ equals ___ in all.

Talk It Over

4. What happens when you join two groups of objects?

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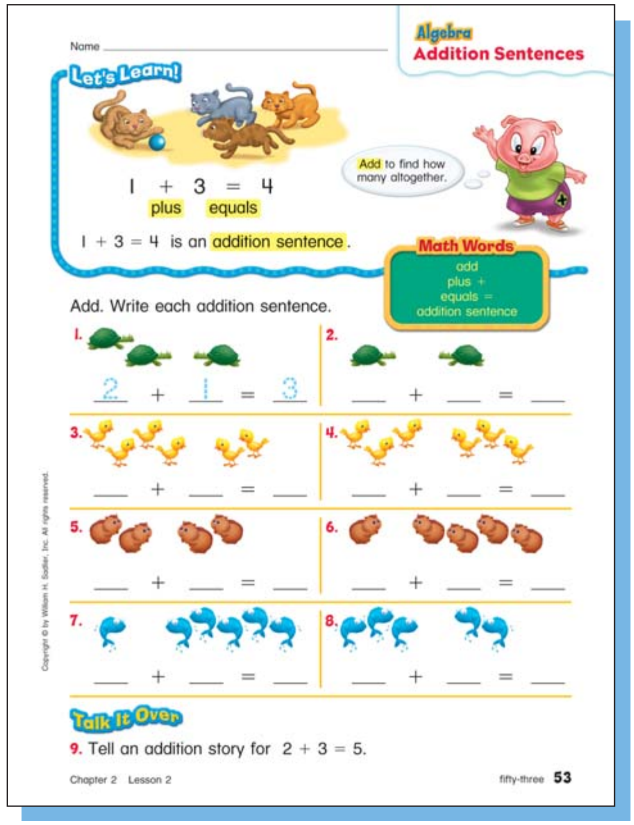
(Concrete Presentation)



Name

Algebra
Addition Sentences

Let's Learn!

1 + 3 = 4
plus equals

Add to find how many altogether.

1 + 3 = 4 is an addition sentence.

Math Words
add
plus +
equals =
addition sentence

Add. Write each addition sentence.

1. 2 + 1 = 3

2. ___ + ___ = ___

3. ___ + ___ = ___

4. ___ + ___ = ___

5. ___ + ___ = ___

6. ___ + ___ = ___

7. ___ + ___ = ___

8. ___ + ___ = ___

Talk It Over

9. Tell an addition story for 2 + 3 = 5.

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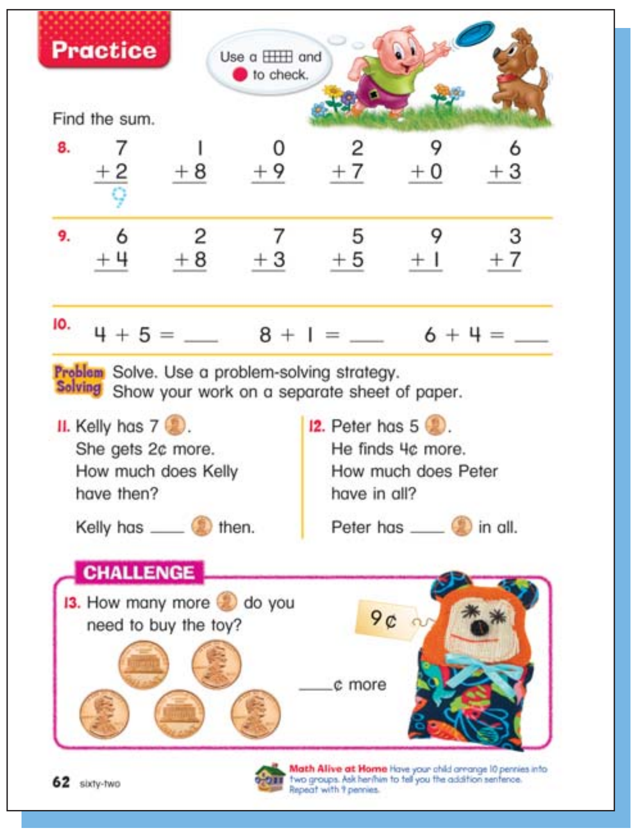
Chapter 2 Lesson 2 fifty-three 53



Practice

Use a and to check.

Find the sum.

8. 7 + 2 = 9, 1 + 8, 0 + 9, 2 + 7, 9 + 0, 6 + 3

9. 6 + 4, 2 + 8, 7 + 3, 5 + 5, 9 + 1, 3 + 7

10. 4 + 5 = ___ 8 + 1 = ___ 6 + 4 = ___

Problem Solving Solve. Use a problem-solving strategy.
Show your work on a separate sheet of paper.

11. Kelly has 7 . She gets 2¢ more. How much does Kelly have then?
Kelly has ___ then.

12. Peter has 5 . He finds 4¢ more. How much does Peter have in all?
Peter has ___ in all.

CHALLENGE

13. How many more do you need to buy the toy?

9¢

___¢ more

Math Alive at Home Have your child arrange 10 pennies into two groups. Ask her/him to tell you the addition sentence. Repeat with 9 pennies.

62 sixty-two

Grade 1 Student Edition pages 53, 62

(Visual and Symbolic Presentations)

3. Fluency in Numerical Operations

Computational fluency in mathematics is achieved through the development of number sense and extensive practice in using efficient and accurate methods for computing. Automaticity in computing efficiently and accurately frees mental energy for problem solving in the same way that automaticity in decoding words, playing scales on a musical instrument, or executing basic movements in a physical sport correctly and quickly advances interpretation and helps improve performance. Moreover, fluency in decoding enables children to read word problems in mathematics carefully, attending fully to the meaning of the text in order to understand the problem itself.

Just as elementary grade students should learn to identify most common and regularly spelled words accurately and quickly without relying on context, so should they also learn how to perform the basic arithmetical operations accurately without a calculator and using the standard algorithms. Most importantly, students should understand why and how the basic algorithms of arithmetic work. This understanding is necessary to support further learning in mathematics. One of the best ways to ensure that students gain this understanding is to defer frequent calculator usage until after they understand the standard algorithms and are fluent with them.

Standard algorithms were gradually developed many centuries ago for their efficiency, accuracy, and generality—that is, they work in all situations. They are theoretically and practically important methods for computing. They contain in their very structure all the basic properties of the base-ten place-value system, set forth in as efficient a manner as possible. An understanding of how and why they work, as well as the ability to use them fluently, provides the foundation for mathematical competence. As children acquire knowledge of the underlying structure of a particular operation and explore different ways to perform it, they should also learn how to use the standard algorithm for the operation. After they learn a standard algorithm for an operation, whatever they then choose to use routinely should be judged on the basis of efficiency and accuracy. Children should be able to explain whatever method they use and see the usefulness of methods that are efficient, accurate, and general.

How research supports it...

Research Reviews: A 15-member group of mathematicians, appointed by the Mathematical Association of America to respond to a set of questions about algorithms and algorithmic thinking posed by the National Council of Teachers of Mathematics Commission on the Future of the Standards, stated that "standard mathematical definitions and algorithms serve as a vehicle of human communication" and that they should be taught to all children (Ross, 1997).

> The starting point for the development of children's creativity and skills should be established concepts and algorithms. … Success in mathematics needs to be grounded in well-learned algorithms as well as understanding of the concepts (Ross, 1998).

Notices of the American Mathematical Society states that "all the algorithms of arithmetic are preparatory for algebra. … The division algorithm is also significant for later understanding of real numbers" (American Mathematical Society Association Resource Group for the NCTM Standards, 1998).

Children in almost all of the highest scoring countries in the Third International Mathematics and Science Survey (TIMSS) do not use calculators as part of mathematics instruction before grade 6. A meta-analysis of studies on calculator use in this country also recommends against its use with young children.

> Because limited research has been conducted featuring the early grades, calculators should be restricted to experimentation and concept development activities (Ellington, 2004).

Research in reading shows that the ability to decode words fluently is highly related to successful reading comprehension.

> Because the ability to obtain meaning from print depends so strongly on the development of word recognition accuracy and reading fluency, both should be regularly assessed in the classroom…(Snow, Burns, & Griffin, 1998).

Individual Studies: A framework for understanding biological and cultural influences on children's cognitive and academic development shows how automatic execution of arithmetical operations is related to problem solving.

> Once procedures are automatized, they require little conscious effort to use, which, in turn, frees attentional and working memory resources for use on other, more important features of the problem (Geary, 1995).

In a study, two mathematicians explain how students learn concepts that are crucial in core applications of mathematics today when they are taught to understand and use the long division algorithm. An understanding of this algorithm prepares them for the next level of mathematical learning. This level includes, above all, the structure of real numbers and what convergence of series and sequences means. It also includes students' ability to construct correct and efficient algorithms on their own, an ability that is best developed by extensive study of examples of sophisticated and powerful algorithms such as the standard long-division algorithm:

> …the long-division algorithm is an essential tool for understanding what a real number is (Klein & Milgram, 2000).

An analysis of the fourth grade results in mathematics on the Third International Mathematics and Science Study finds that use of calculators in U.S. fourth grade mathematics classes is about twice the international average.

> U.S. fourth graders use calculators and computers in mathematics class more frequently than do students in most other TIMSS countries. ...in six of the seven nations that outscore the U.S. in mathematics, teachers of 85% or more of the students report that students never [or hardly ever] use calculators in class (National Center for Education Statistics, 2000).

A study of calculator use and NAEP math assessment data found that fourth graders who took the 1996 NAEP math assessment and reported using calculators every day scored the lowest on the math test. A similar correlation was noted in the TIMSS data.

> On [NAEP and TIMSS] tests, students are asked how often they use calculators in class. And on both tests, calculator use is correlated with lower math scores (Loveless, 2000).

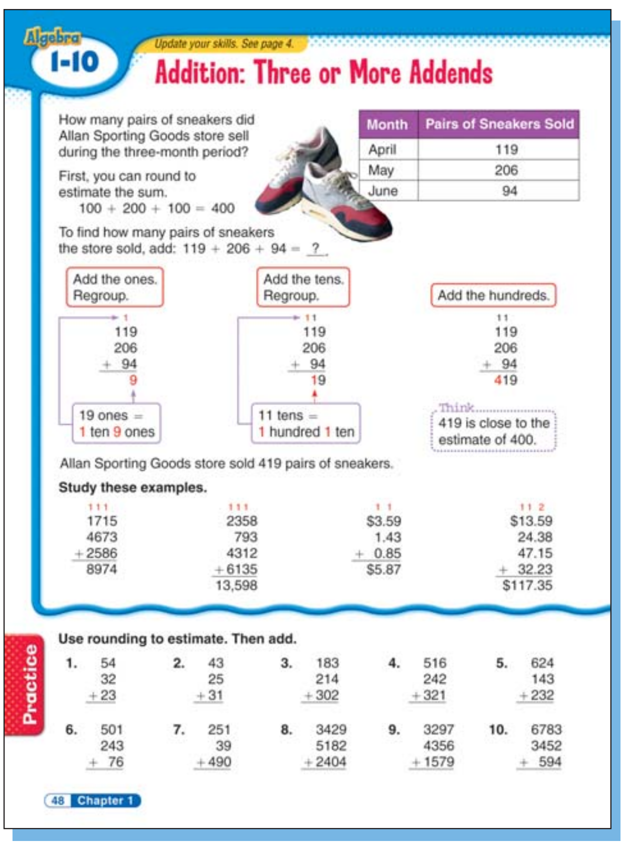
Research in reading also shows the value of fluency. Research shows that children who are dependent on context for word identification are slower, poorer readers. Fluent readers can pay more attention to what the text means because they do not have to concentrate on decoding the words.

> Slow, capacity-draining word-recognition processes require cognitive resources that should be allocated to higher-level processes of text integration and comprehension. Thus, reading for meaning is hindered, unrewarding reading experiences multiply, and practice is avoided or merely tolerated without real cognitive involvement (Stanovich, 1986).

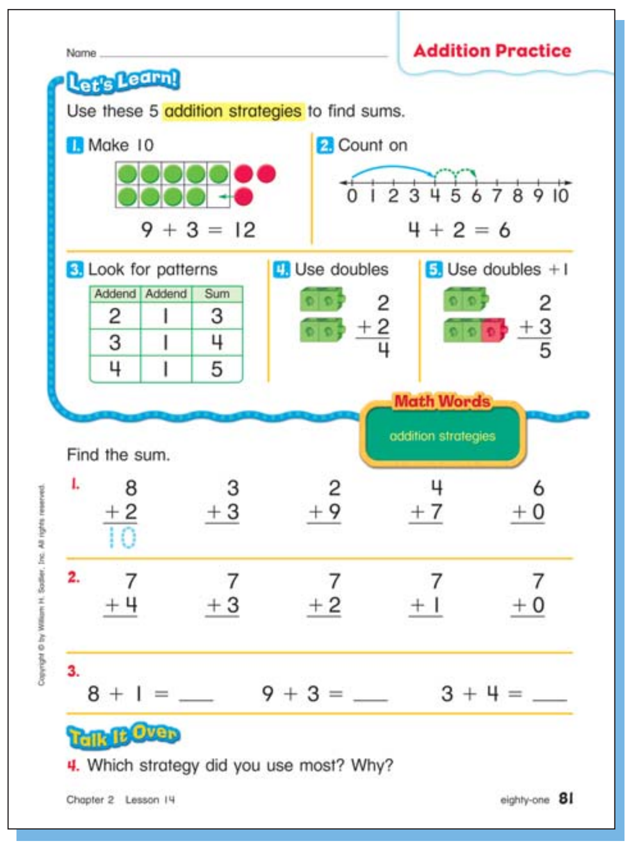
> Automaticity is vital in education because it allows us to become more skillful in mental tasks (Willingham, 2004).

Progress in Mathematics responds with...

- **Step-by-step** algorithms that allow children to quickly and accurately perform mathematical operations, without relying on the use of calculators
- Ample **Practice** of strategies and facts in order to generate fluency



Grade 5 Student Edition page 48 **(Step-by-Step)**



Grade 1 Student Edition page 81 **(Practice)**

4. Problem Solving

Problem solving in a mathematics class is an activity in which students both develop and apply their reading skills, conceptual knowledge, and computational skills to solve a mathematical problem in which the answer is not evident. Such problems are not repetitive exercises of a skill just taught in a lesson. They may require use of previously learned knowledge and skills, recently learned knowledge and skills, and/or the simultaneous application of multiple skills and concepts. In solving problems, students analyze the information given in a problem, consider the range of strategies they know for solving the problem, decide on the strategies that best address the problem as they have analyzed it, or develop new strategies for solving the problem. Students acquire flexibility in using the strategies they know or in developing new strategies through practice in solving problems in a variety of formats and contexts and at increasing levels of complexity. Graphic and semantic organizers as well as mental imagery can be helpful tools to examine and represent relationships in a problem. Successful problem solvers have the reading skills, conceptual knowledge, and computational fluency required for solving the problems their teachers or textbooks present to them.

How research supports it...

Research Reviews: Skill in spatial representation and reading as well as judgment in using mathematical knowledge are emphasized in a reference to the research on problem solving.

> Strong mathematical problem solving appears to be associated with, among others, the ability to spatially represent mathematical relations, the ability to translate word problems into appropriate equations, and an understanding of how and when to use mathematical equations (Geary, 1995).

A review of research studies on problem solving in science and mathematics education stresses the teacher's role as well as background knowledge.

> Our reading of a representative group of studies emphasizing the development of reasoning skills through inquiry-based problem solving suggests that: (1) students always need teacher guidance in solving problems, and (2) reasoning ability depends as much on content knowledge and its manipulation as on the students' presumed level of intellectual development…(Gross & Stotsky, 2000).

A review of the use of graphic organizers in reading and across the other curriculum areas finds "solid evidence for the effectiveness of graphic organizers in facilitating learning" when students are explicitly instructed in how to use them (Hall & Strangman, 2001).

Individual Studies: A study of an "explicit strategy method" for teaching fourth graders how to translate a word problem into mathematical form suggests the effectiveness of direct instruction in problem solving.

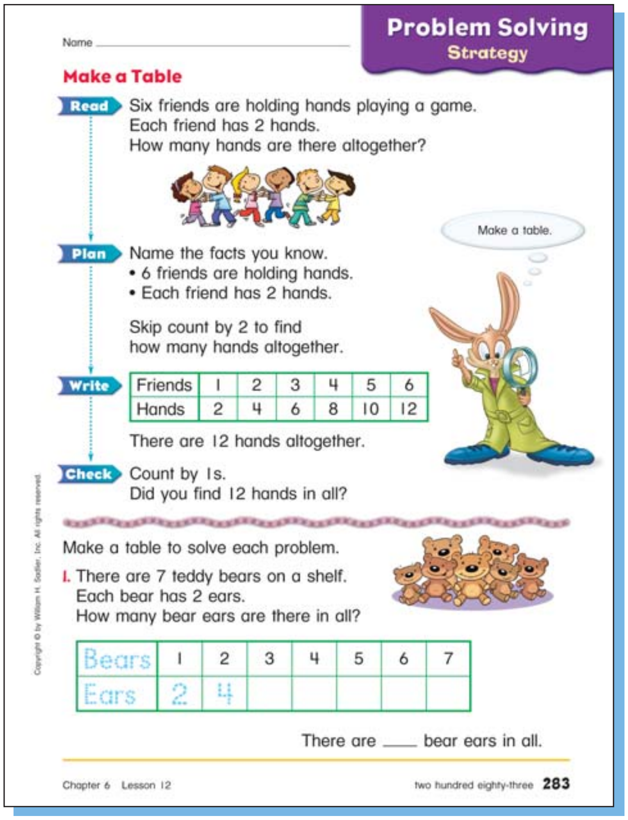
> In order to translate a word problem into mathematical form, this study supports the position that a program constructed to teach prerequisite skills in a sequential manner and, more importantly, explicitly model and teach each step in the translation process is significantly more effective than "guided discovery" methods, especially for lower performing students (Darch, Carnine, & Gersten, 1984).

A study of the relation between conceptual knowledge and procedural knowledge suggests the importance of making sure children visualize a problem correctly.

> …supporting the correct representation of problems is an effective tool for improving problem-solving knowledge (Rittle-Johnson, Siegler, & Alibali, 2001).

Progress in Mathematics responds with…

- **Step-by-step** strategy development in which each step of the problem-solving process is clearly modeled
- Opportunities for **analysis and review** in which children analyze information and apply previously learned strategies to solving problems with different number types



Problem Solving
Strategy

Name ____________________

Make a Table

Read Six friends are holding hands playing a game.
Each friend has 2 hands.
How many hands are there altogether?

Plan Name the facts you know.
• 6 friends are holding hands.
• Each friend has 2 hands.

Skip count by 2 to find how many hands altogether.

Write

Friends	1	2	3	4	5	6
Hands	2	4	6	8	10	12

There are 12 hands altogether.

Check Count by 1s.
Did you find 12 hands in all?

Make a table to solve each problem.

1. There are 7 teddy bears on a shelf.
Each bear has 2 ears.
How many bear ears are there in all?

Bears	1	2	3	4	5	6	7
Ears	2	4					

There are ____ bear ears in all.

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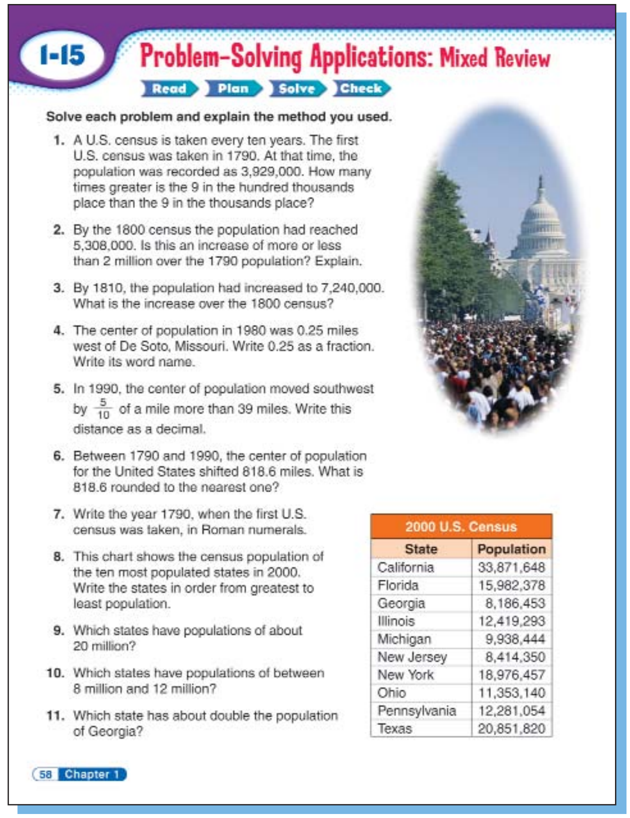
Grade 1 Student Edition page 283 **(Step-by-Step)**



1-15 Problem-Solving Applications: Mixed Review

Read Plan Solve Check

Solve each problem and explain the method you used.

1. A U.S. census is taken every ten years. The first U.S. census was taken in 1790. At that time, the population was recorded as 3,929,000. How many times greater is the 9 in the hundred thousands place than the 9 in the thousands place?
2. By the 1800 census the population had reached 5,308,000. Is this an increase of more or less than 2 million over the 1790 population? Explain.
3. By 1810, the population had increased to 7,240,000. What is the increase over the 1800 census?
4. The center of population in 1980 was 0.25 miles west of De Soto, Missouri. Write 0.25 as a fraction. Write its word name.
5. In 1990, the center of population moved southwest by $\frac{5}{10}$ of a mile more than 39 miles. Write this distance as a decimal.
6. Between 1790 and 1990, the center of population for the United States shifted 818.6 miles. What is 818.6 rounded to the nearest one?
7. Write the year 1790, when the first U.S. census was taken, in Roman numerals.
8. This chart shows the census population of the ten most populated states in 2000. Write the states in order from greatest to least population.
9. Which states have populations of about 20 million?
10. Which states have populations of between 8 million and 12 million?
11. Which state has about double the population of Georgia?

2000 U.S. Census

State	Population
California	33,871,648
Florida	15,982,378
Georgia	8,186,453
Illinois	12,419,293
Michigan	9,938,444
New Jersey	8,414,350
New York	18,976,457
Ohio	11,353,140
Pennsylvania	12,281,054
Texas	20,851,820

58 Chapter 1

Grade 5 Student Edition page 58 **(Analysis and Review)**

5. Vocabulary Development

Vocabulary development is one of the basic building blocks in learning and promoting math literacy. Understanding the exact meaning of subject-specific words and using them correctly is crucial to mathematical comprehension. Children must learn the words that they are likely to see—and hear—again and again in their mathematics lessons—the words that express basic mathematical relationships, terms, and shapes. Such relational phrases as "bigger than," "smaller than," "equal to," "less than," and "fewer than"; such terms as "fraction," "right angle," "multiplication," "addition," and "subtraction"; and such shapes as "triangle," "quadrilateral," "cone," "cylinder," "polygon," and "hexagon" need to be defined, exemplified, and used precisely in mathematical contexts.

Although a list of such words or phrases in elementary arithmetic is not large, and children seem to learn many of them through daily discussion and repeated exposure in their textbooks, children often misunderstand them at a basic level. Teachers should give students exact definitions of mathematical terms and shapes when they are learning them, expect these words to be used correctly, and give students practice in the correct use of relational phrases.

Instruction in a mathematics vocabulary might profitably be differentiated and separated from instruction in a general reading vocabulary. As with scientific terms, mathematical terms have a specific and consistent meaning uninfluenced by context. This is not at all the case with the words children encounter in literary reading or in their everyday experiences. For the common prefixes, suffixes, and word roots that appear in a large number of related words in mathematics and science (for example, the prefix in millisecond, millimeter, and millennium), structural analysis can also be a productive approach. Children's understanding of their basic mathematical vocabulary should be regularly monitored.

The crucial concept of place value that underlies all of arithmetic and most of algebra can also be developed in counting games in beginning arithmetic if children are taught to read numbers aloud in a way that treats zero as a number to be read, not as a place-holder, thus saying all places: for example, reading *205* aloud as "two one-hundreds plus no, or zero, tens plus five ones." This kind of vocabulary practice in beginning arithmetic may also help children avoid confusion when they hear a number read aloud in English in the common way—the way they are taught to read numbers aloud in reading class (for example, reading *207* aloud as "two hundred and seven")—if their teachers point out the difference between the common way of reading numbers aloud in English and a mathematically precise way of reading them aloud.

For children who are just learning the English language, mathematics with its wide use of manipulatives, illustrations, and graphics is the optimal subject for conveying language and content simultaneously. Teaching a mathematical vocabulary to these students, especially relational phrases, is a crucial first step in developing the students' ability to read and understand word problems later. It may be necessary to give English language learners more than the usual amount of direct instruction and practice on explicit vocabulary development.

How research supports it...

Research Reviews: The authors of a comprehensive review of the research on vocabulary instruction offer a number of recommendations to teachers regarding vocabulary instruction. Among them are the following:

> (1) Include instruction in both specific-word and transferable and generalizable strategies. (2) Provide struggling readers a systematic and sustained program of vocabulary instruction that teaches them more important words and efficient strategies. And (3) select suitable strategies from a range of empirically validated instructional procedures that are compatible with your instructional objectives (Bauman, Kame'enui, & Ash, 2003).

Another review of research on vocabulary instruction highlights the effects of direct instruction in the content areas.

> Direct instruction on words that are critical to new content produces the most powerful learning. The effects of vocabulary instruction are even more powerful when the words selected are those that students most likely will encounter when they learn new content (Marzano, Pickering, & Pollock , 2001).

Individual Studies: Young Asian students' higher mathematical learning may be related to the regularity of spoken number words in East Asian languages. For example, Korean elementary school children aged 6, 7, and 8 carry out 2- and 3-digit addition and subtraction considerably more accurately than do their U.S. age-mates.

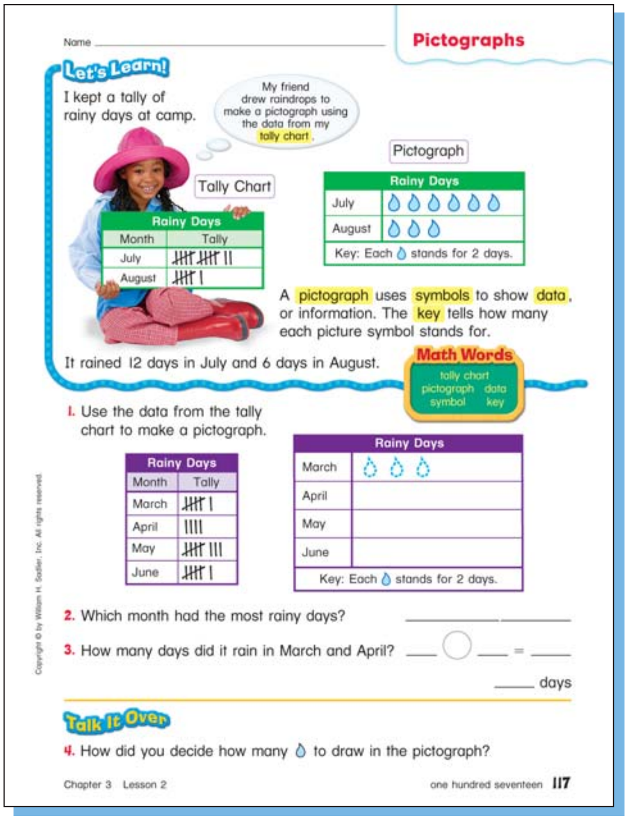
> Korean children do not have to find the tens and the ones for a given 2-digit number; they are already given in the Korean number words. This example underscores the special difficulties imposed on English-speaking children and suggests that it might be helpful for them to use "tens words"—English versions of Korean words—to support their multiunit thinking (Fuson & Kwon, 1992).

A student who can read a key word in mathematics correctly may not necessarily understand what the word means mathematically. A reading researcher offers three principles for effective vocabulary instruction:

> Principle 1: give both context and definitions; Principle 2: encourage 'deep' processing; and Principle 3: give multiple exposures (Stahl, 1986).

Progress in Mathematics responds with...

- **Math Words** that teach children the language of mathematics and how to communicate mathematically
- **Talk It Over** activities that connect children's understanding of math concepts with math vocabulary
- **Do You Remember?** exercises that review previously learned math vocabulary
- **Math Vocabulary** activities (in the Teacher's Edition) that engage children in discussions using math vocabulary
- **Write About It** activities that encourage children to use math vocabulary in their writing (not shown)
- An **On-line Grade Level Glossary** that illustrates and defines program math vocabulary (not shown)


Name ______ **Pictographs**

Let's Learn!

I kept a tally of rainy days at camp.

My friend drew raindrops to make a pictograph using the data from my tally chart.

Tally Chart

Rainy Days	
Month	Tally
July	𝍸𝍸 II
August	𝍸 I

Pictograph

Rainy Days	
July	💧💧💧💧💧💧
August	💧💧💧
Key: Each 💧 stands for 2 days.	

A pictograph uses symbols to show data, or information. The key tells how many each picture symbol stands for.

It rained 12 days in July and 6 days in August.

Math Words: tally chart, pictograph, data, symbol, key

1. Use the data from the tally chart to make a pictograph.

Rainy Days	
Month	Tally
March	𝍸 I
April	IIII
May	𝍸 III
June	𝍸 I

Rainy Days	
March	💧💧💧
April	
May	
June	
Key: Each 💧 stands for 2 days.	

2. Which month had the most rainy days? ______
3. How many days did it rain in March and April? ___ ◯ ___ = ___ ___ days

Talk It Over

4. How did you decide how many 💧 to draw in the pictograph?

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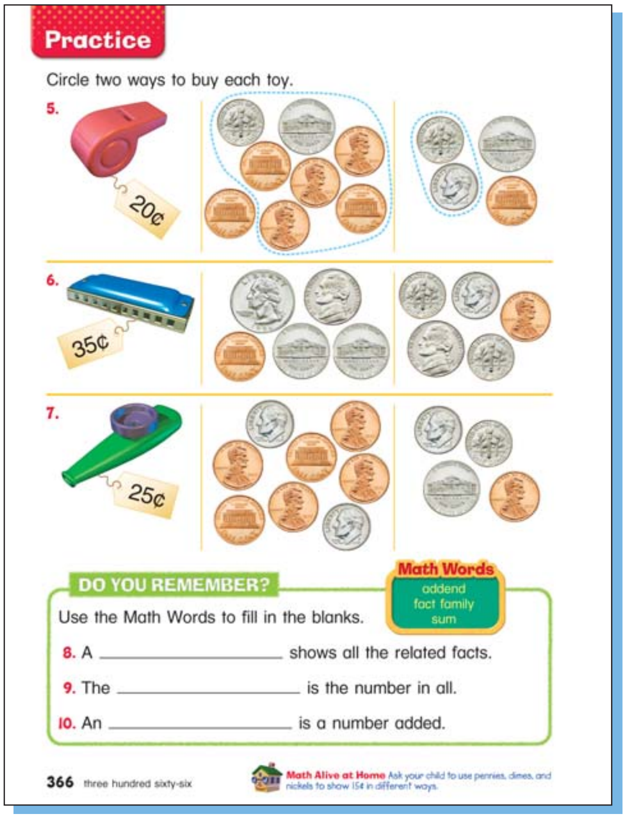
Chapter 3 Lesson 2 one hundred seventeen 117

Grade 2 Student Edition page 117
(Math Words, Talk It Over)


Practice

Circle two ways to buy each toy.

5. 20¢
6. 35¢
7. 25¢

DO YOU REMEMBER?

Math Words: addend, fact family, sum

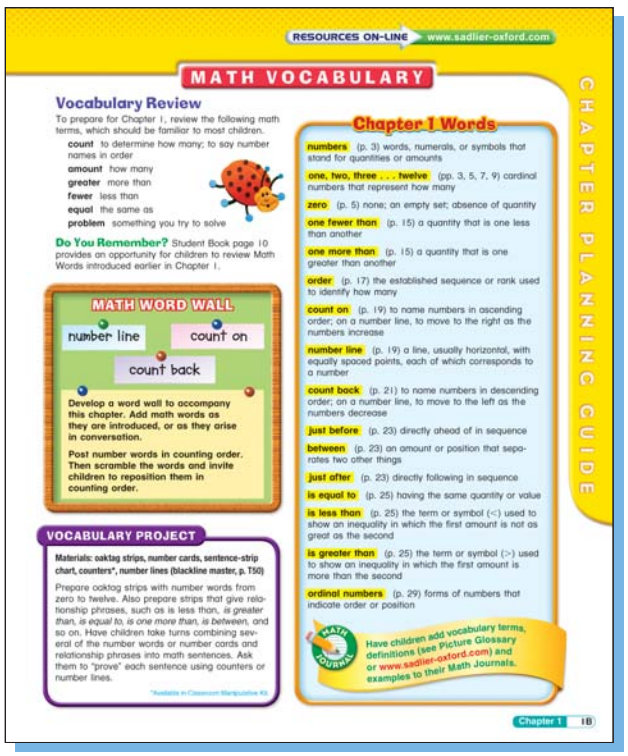
Use the Math Words to fill in the blanks.

8. A ______ shows all the related facts.
9. The ______ is the number in all.
10. An ______ is a number added.

366 three hundred sixty-six

Math Alive at Home Ask your child to use pennies, dimes, and nickels to show 15¢ in different ways.

Grade 1 Student Edition page 366 **(Do You Remember?)**


RESOURCES ON-LINE www.sadlier-oxford.com

MATH VOCABULARY

Vocabulary Review

To prepare for Chapter 1, review the following math terms, which should be familiar to most children.

count to determine how many; to say number names in order
amount how many
greater more than
fewer less than
equal the same as
problem something you try to solve

Do You Remember? Student Book page 10 provides an opportunity for children to review Math Words introduced earlier in Chapter 1.

MATH WORD WALL

number line | count on | count back

Develop a word wall to accompany this chapter. Add math words as they are introduced, or as they arise in conversation.

Post number words in counting order. Then scramble the words and invite children to reposition them in counting order.

VOCABULARY PROJECT

Materials: oaktag strips, number cards, sentence-strip chart, counters*, number lines (blackline master, p. T50)

Prepare oaktag strips with number words from zero to twelve. Also prepare strips that give relationship phrases, such as is less than, *is greater than, is equal to, is one more than, is between*, and so on. Have children take turns combining several of the number words or number cards and relationship phrases into math sentences. Ask them to "prove" each sentence using counters or number lines.

*Available in Classroom Manipulative Kit

Chapter 1 Words

numbers (p. 3) words, numerals, or symbols that stand for quantities or amounts
one, two, three . . . twelve (pp. 3, 5, 7, 9) cardinal numbers that represent how many
zero (p. 5) none; an empty set; absence of quantity
one fewer than (p. 15) a quantity that is one less than another
one more than (p. 15) a quantity that is one greater than another
order (p. 17) the established sequence or rank used to identify how many
count on (p. 19) to name numbers in ascending order; on a number line, to move to the right as the numbers increase
number line (p. 19) a line, usually horizontal, with equally spaced points, each of which corresponds to a number
count back (p. 21) to name numbers in descending order; on a number line, to move to the left as the numbers decrease
just before (p. 23) directly ahead of in sequence
between (p. 23) an amount or position that separates two other things
just after (p. 23) directly following in sequence
is equal to (p. 25) having the same quantity or value
is less than (p. 25) the term or symbol (<) used to show an inequality in which the first amount is not as great as the second
is greater than (p. 25) the term or symbol (>) used to show an inequality in which the first amount is more than the second
ordinal numbers (p. 29) forms of numbers that indicate order or position

MATH JOURNAL Have children add vocabulary terms, definitions (see Picture Glossary or www.sadlier-oxford.com) and examples to their Math Journals.

CHAPTER PLANNING GUIDE

Chapter 1 1B

Grade 1 Teacher's Edition page 1B **(Math Vocabulary)**

6. Practice and Review

Practice and review are fundamental to mathematical learning. They are as necessary in mathematics as they are in music and sports for learning basic skills or procedures, for mastering them, and for making them automatic. Regular practice leads not only to greater fluency in the use of a skill but also to better retention of the skill. Indeed, practice beyond the point of mastery is recommended for a new skill to become long-lasting. There are, however, different purposes for practice.

- At first, practice should be guided to ensure that students are practicing correctly what they have just learned.
- Practice should then be differentiated to provide a challenge to students who learn quickly or are at advanced levels of skill development, and to provide reinforcement to students who learn at a slower pace or demonstrate weaknesses in the new skill.
- Practice should then be undertaken independently.
- Finally, practice should be incorporated in reviews that are intended to sustain fluency in skills that have been mastered. Sustained practice, or ongoing review, may mean using newly learned computational skills to solve increasingly more complex math problems. It may also mean doing short daily exercises in the form of mental math or taking regular quizzes that incorporate material learned earlier in the year.

How research supports it...

Research Reviews: In a discussion of the role of basic skills in mathematical learning, a large number of studies are cited as support for the importance of practice for skill retention.

> …The argument that drill and practice and the development of basic cognitive skills, such as fact retrieval, are unnecessary and unwanted in mathematics education fails to appreciate the importance of basic skills for mathematical development (Geary, 1994).

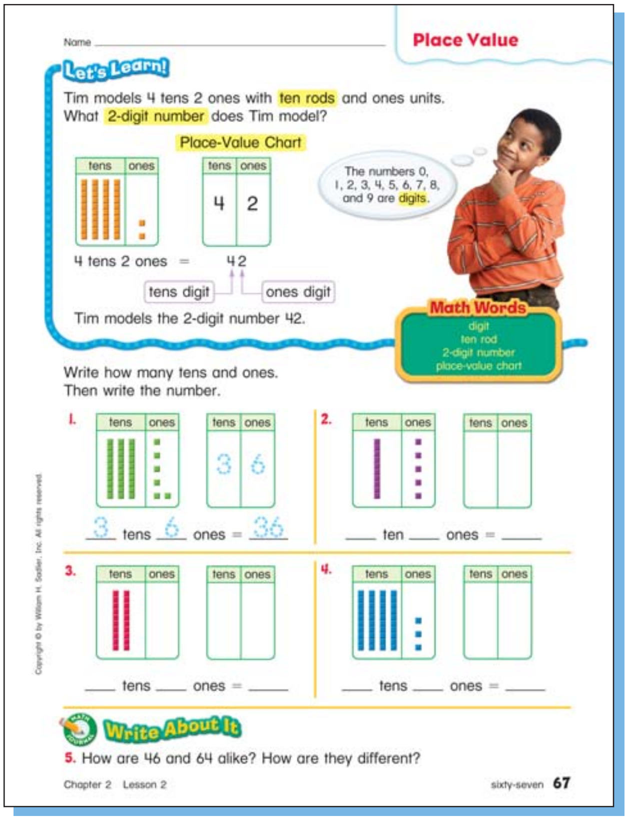
Individual Studies: Many individual studies emphasize the value of practice.

> That students will only remember what they have extensively practiced—and that they will only remember for the long term that which they have practiced in a sustained way over many years—are realities that can't be bypassed (Willingham, 2004).

> Procedural learning requires extensive practice on the whole range of problems on which the procedure might eventually be used (Geary, 1995).

Progress in Mathematics responds with...

- **Guided practice** that provides instructional support so children can transition to independent practice
- An abundance of **independent practice** opportunities allows children to achieve mastery of new skills as they work on their own
- **Do You Remember?** exercises that provide spiral or mixed review of previously learned skills
- **Cumulative Review** tests that review previous chapter materials
- **Lesson Readiness** activities (in the Teacher's Edition) that review skills and help sustain fluency
- **Mental Math** and **Problem of the Day** exercises that provide daily maintenance



Name ____________________

Place Value

Let's Learn!

Tim models 4 tens 2 ones with ten rods and ones units.
What 2-digit number does Tim model?

Place-Value Chart

Tim models the 2-digit number 42.

Math Words
digit
ten rod
2-digit number
place-value chart

Write how many tens and ones.
Then write the number.

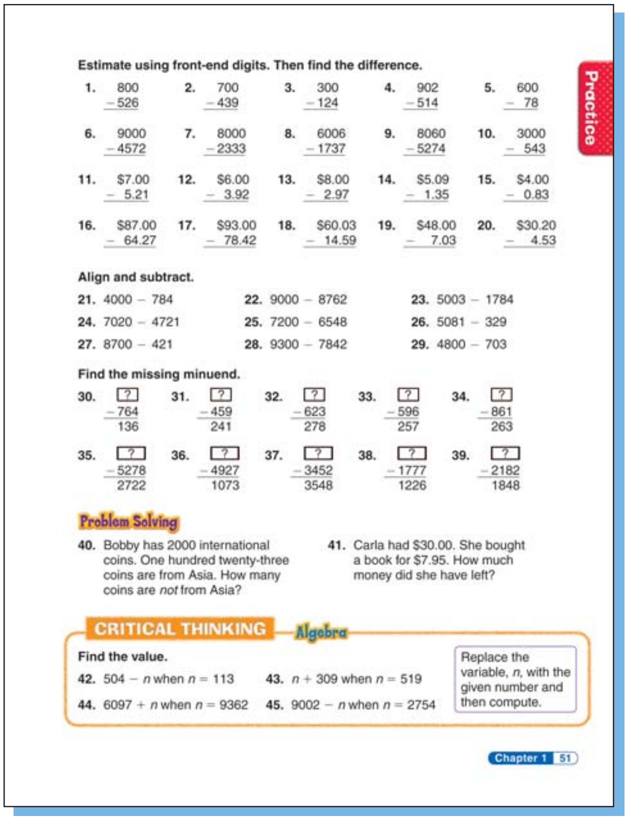
1. 3 tens 6 ones = 36
2. ___ ten ___ ones = ___
3. ___ tens ___ ones = ___
4. ___ tens ___ ones = ___

Write About It

5. How are 46 and 64 alike? How are they different?

Chapter 2 Lesson 2 sixty-seven 67

Grade 2 Student Edition page 67 **(Guided Practice)**



Practice

Estimate using front-end digits. Then find the difference.

1. 800 − 526	2. 700 − 439	3. 300 − 124	4. 902 − 514	5. 600 − 78
6. 9000 − 4572	7. 8000 − 2333	8. 6006 − 1737	9. 8060 − 5274	10. 3000 − 543
11. $7.00 − 5.21	12. $6.00 − 3.92	13. $8.00 − 2.97	14. $5.09 − 1.35	15. $4.00 − 0.83
16. $87.00 − 64.27	17. $93.00 − 78.42	18. $60.03 − 14.59	19. $48.00 − 7.03	20. $30.20 − 4.53

Align and subtract.

21. 4000 − 784	22. 9000 − 8762	23. 5003 − 1784
24. 7020 − 4721	25. 7200 − 6548	26. 5081 − 329
27. 8700 − 421	28. 9300 − 7842	29. 4800 − 703

Find the missing minuend.

30. ? − 764 136	31. ? − 459 241	32. ? − 623 278	33. ? − 596 257	34. ? − 861 263
35. ? − 5278 2722	36. ? − 4927 1073	37. ? − 3452 3548	38. ? − 1777 1226	39. ? − 2182 1848

Problem Solving

40. Bobby has 2000 international coins. One hundred twenty-three coins are from Asia. How many coins are *not* from Asia?

41. Carla had $30.00. She bought a book for $7.95. How much money did she have left?

CRITICAL THINKING Algebra

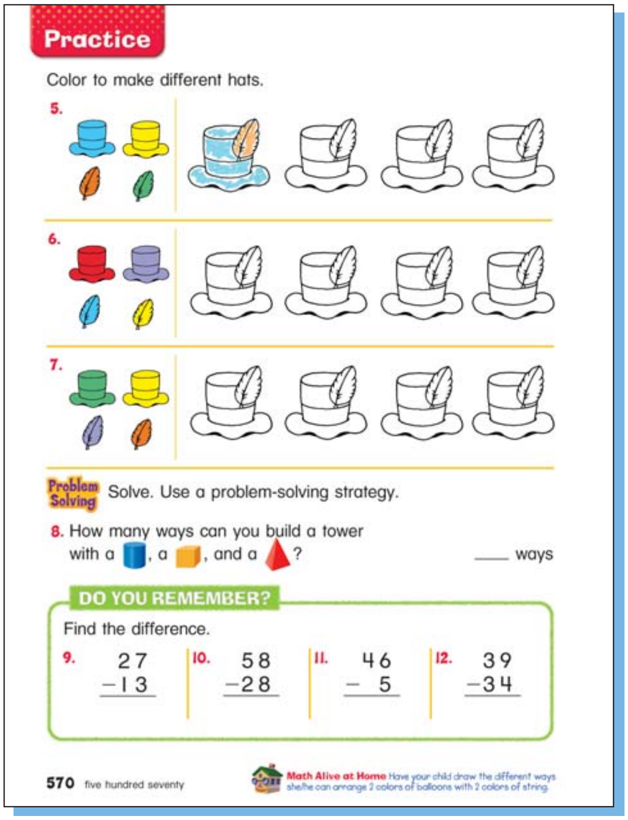
Find the value.

42. $504 - n$ when $n = 113$
43. $n + 309$ when $n = 519$
44. $6097 + n$ when $n = 9362$
45. $9002 - n$ when $n = 2754$

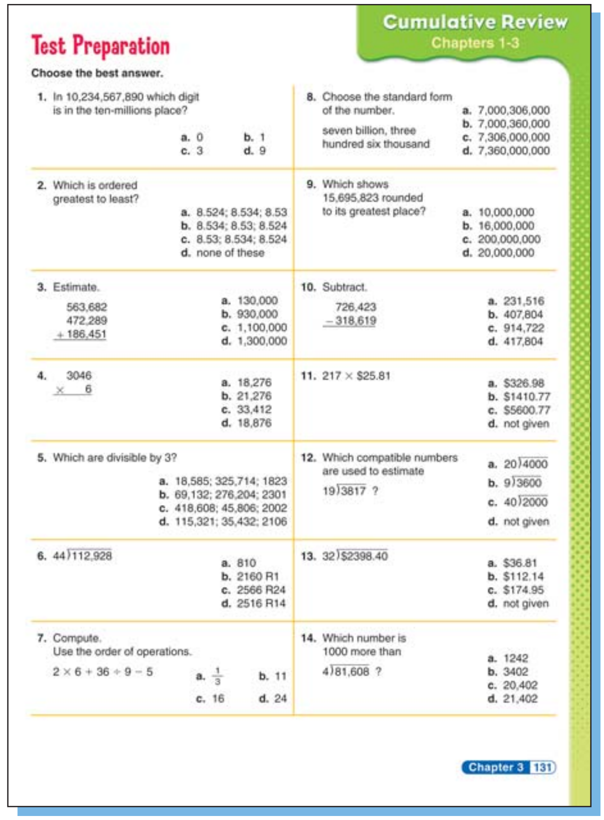
Replace the variable, *n*, with the given number and then compute.

Chapter 1 51

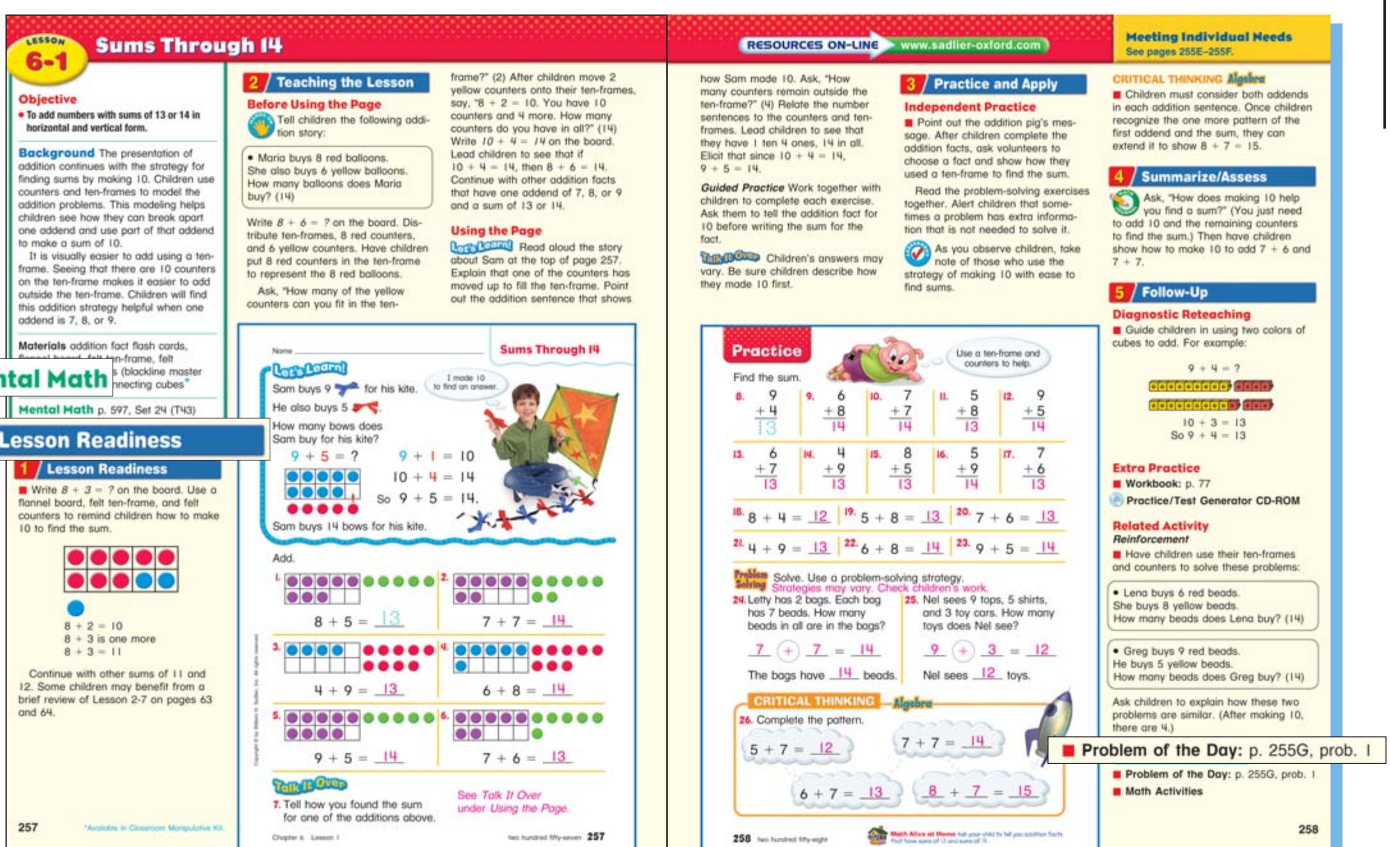
Grade 5 Student Edition page 51 **(Independent Practice)**



Grade 1 Student Edition page 570 **(Do You Remember?)**



Grade 3 Student Edition page 131 **(Cumulative Review)**



Grade 1 Teacher's Edition pages 257– 258

7. Formative and Summative Assessments

A comprehensive review of the current state of assessment theory, knowledge, and practice by the Center for Education sets forth the three broad purposes of assessment in the following order: to assist learning, to measure individual achievement, and to evaluate instructional programs. Assessment should be an integral part of mathematics instruction. It can guide instruction as well as inform students of their progress and achievement in mathematical learning. Assessments can take the form of quizzes or other short informal tests, formal tests, teacher questioning and observations, performance tasks, and portfolios showing collections of work samples. Formative or ongoing classroom assessments provide information on individual progress in mathematical learning, helping teachers to make needed adjustments in instruction. Summative assessments provide information on student achievement in relation to external standards or benchmarks.

How research supports it...

Research Reviews: A review of advances in the cognitive sciences and measurement by the Center for Education recommends:

> ...an increased emphasis on classroom formative assessment designed to assist learning... (Center for Education, 2001).

A collection of research-based essays on learning notes two key principles of assessment.

> The key principles of assessment are that they should provide opportunities for feedback and revision and that what is assessed must be congruent with one's learning goals (Bransford, Brown, & Cocking, 2000).

A review of high-quality studies in mathematics education by the Director of the Institute of Education Sciences highlights the value of frequent assessments.

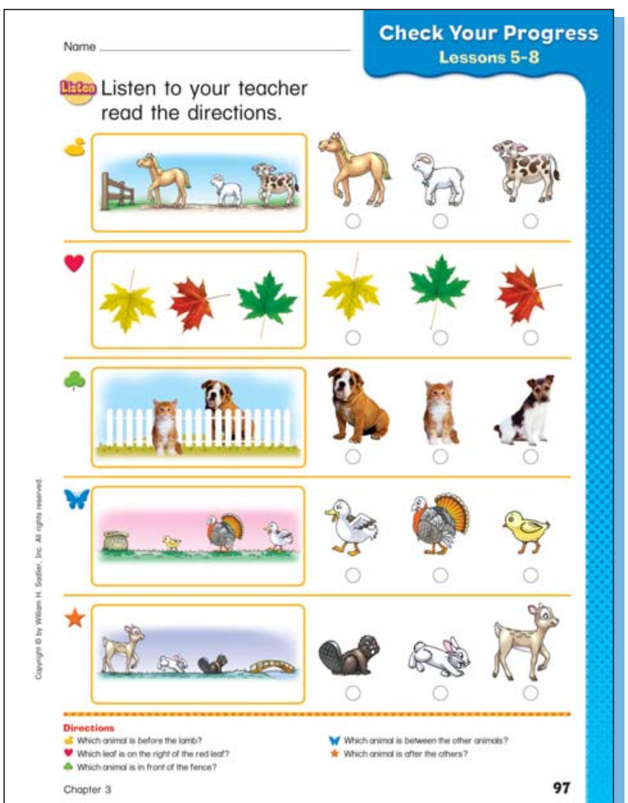
> We know that at the classroom level, frequent assessment is useful, particularly when teachers are given help on what they should do for children who aren't performing well (Whitehurst, 2003).

Individual Studies: Several researchers note that formal standards-based assessments in mathematics should include enough high-level content to assess the full range of student achievement adequately.

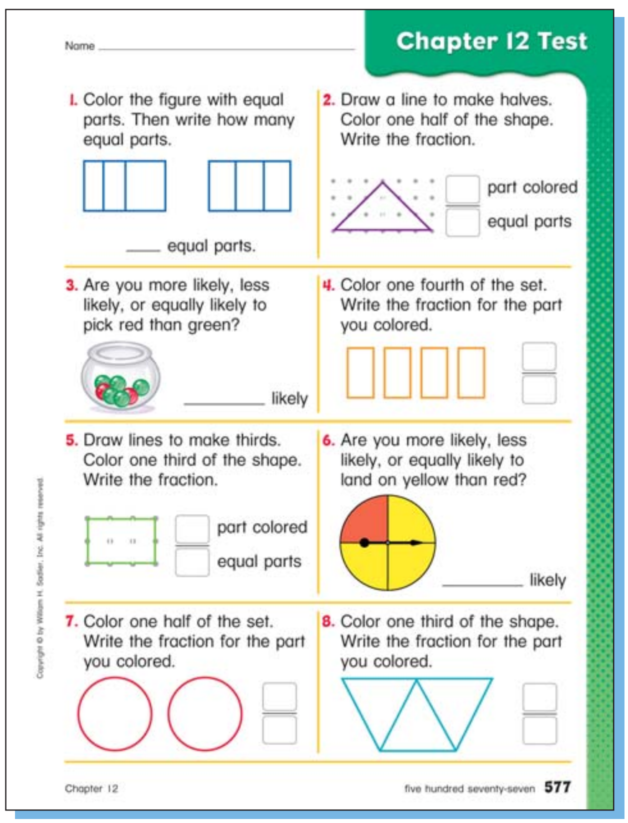
> Much depends on the specificity and difficulty level of the standards themselves at each grade level assessed, as well as the design of the assessments and related reporting mechanisms. Tests that emphasize low-level content may not stimulate significant overall student improvement and may even mask a lack of high-level performance (Clopton, Bishop, & Klein, 2000).

Progress in Mathematics responds with…

- **Formative or ongoing classroom assessments** that continuously provide information about children's abilities so teachers can adjust instruction. Formative assessments include *Diagnostic Pretest, Check Your Progress,* and *Cumulative Review*
- **Summative assessments** that provide information about the extent to which instructional goals have been met. Summative assessments include *Chapter Test, Performance Assessment, Student Test Booklet,* and *Portfolios*
- **Reteaching suggestions** (in the Teacher's Edition) for those children needing extra support



Grade K Student Edition page 97



Grade 1 Student Edition page 577

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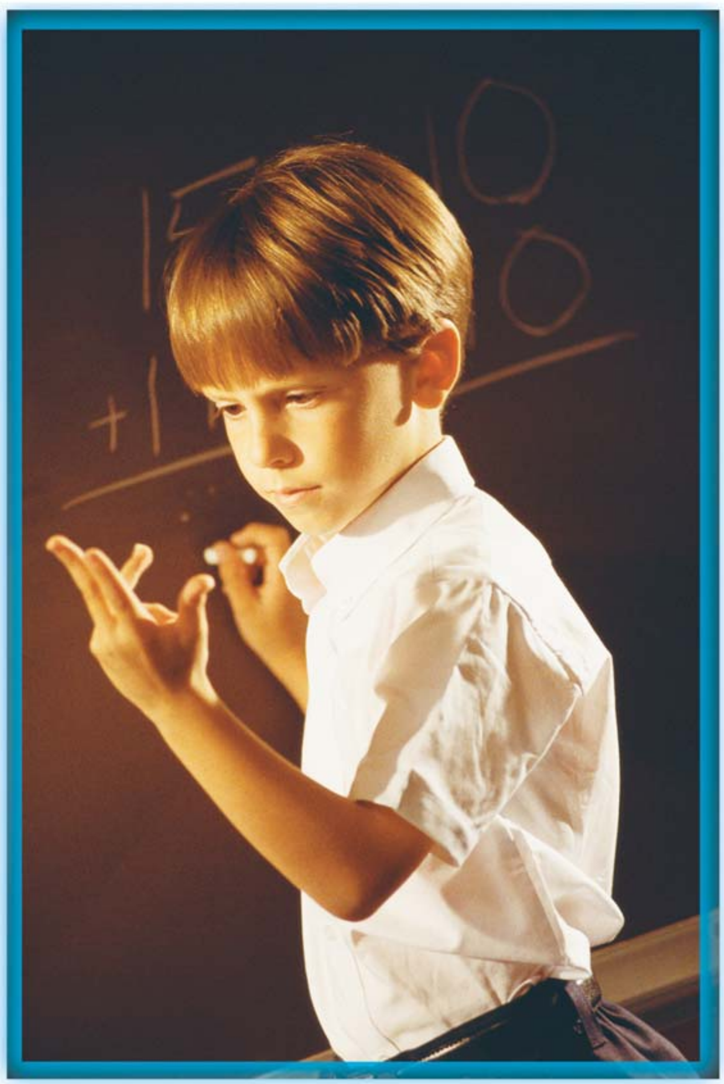
Proven elements of current research are integrated with a successful traditional teaching approach in a comprehensive, self-contained program that supports today's standards.

Progress in Mathematics promotes an excitement for learning and encourages growth in mathematics.



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