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for

Fundamentals of Algebra
Foundations of Algebra
Algebra 1

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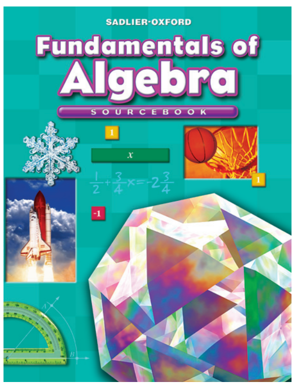
RESEARCH BASE REPORT

for

Middle School Mathematics*

(with Algebra 1)



Fundamentals of Algebra (Course I)

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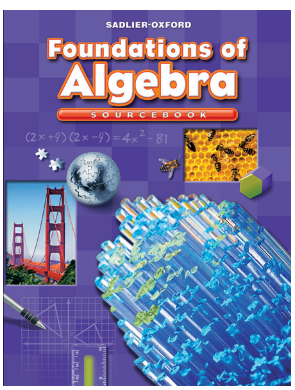

Foundations of Algebra (Course II)

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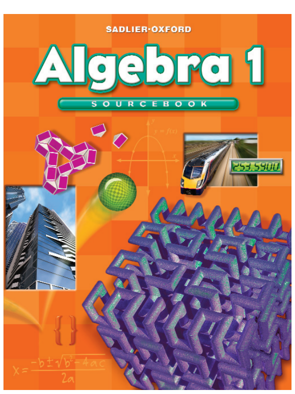

Algebra 1

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*Part of the *Progress in Mathematics* K–8+ Program

Sadlier Mathematics Advisory Board

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Contents

The National Mathematics Advisory Panel (NMAP) was created by President Bush in April 2006, and it was asked to "use the best available scientific research to advise on improvements in the mathematics education of the nation's children." Over a period of nearly two years, the Panel heard public testimony and worked in task groups to fulfill its charge. In March 2008, the Panel issued its final report, entitled *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. This report includes the Panel's main findings and recommendations under the headings "Curricular Content," "Learning Processes," "Teachers and Teacher Education," "Instructional Practices," "Instructional Materials," "Assessment," and "Research Policies and Mechanisms." The recommendations of the NMAP reflect the professional judgment of mathematicians, the results of comparative curriculum studies, and the evidence from large and sound bodies of high quality research studies as defined by demanding criteria.

One purpose of this paper is to show that the major curricular and programmatic features of Sadlier's Middle School Mathematics program represent the best professional knowledge as well as the best practices in mathematics instruction suggested by the findings of independent research studies and meta-analyses of the data from many studies. We do so by referring to:

1. Research reviews and individual studies whose evidence supports the programmatic features of Sadlier's Middle School Mathematics program;
2. Conclusions and recommendations in the final report of the NMAP, which are based largely on the evidence from many research studies and the professional judgment of mathematicians.

For each program feature, we first note how Sadlier's Middle School Mathematics program reflects the conclusions and recommendations of these studies and reports.

A second purpose of this paper is to describe the Product Development Research that Sadlier undertook in order to solicit the views of the mathematics education community: classroom teachers, mathematics supervisors, principals, authors, and national experts in mathematics education. These educators provided comments on each of the titles that are part of Sadlier's Middle School Mathematics program: *Fundamentals of Algebra, Foundations of Algebra*, and *Algebra 1*.

1. Curricular Features

Textbook Organization

Sadlier's Middle School Mathematics program provides:

- middle school SourceBooks that are about half the length of competing texts, with a companion Practice Book;
- a coherent list of priority lessons that can be taught in 140 days;
- an illustration plan that avoids using unnecessary social art to create visual interest and instead relies on diagrams, tables, and graphs to support mathematical understanding;
- inclusion of content from the natural and social sciences only when it serves to focus student attention on the relevant mathematical connections;
- multiple, specific objectives provided on the student page that serve as an advance organizer of the content of the lesson;
- succinct explanations accompanied by diagrams that facilitate predictable teaching time for each concept;
- an instructional design that is designed to facilitate a smooth flow, highlight key concepts as they are reached, and stress readability.

Why?

In an effort to address the diverse standards of different states, textbooks have expanded to the point where the content of an individual textbook usually cannot be taught within a school year. The result is that many teachers devote too little attention to key topics in an effort to address all topics. Additionally, the inordinate length of textbooks is often the result of unnecessary illustrations or nonmathematical content. The NMAP recommends that textbook publishers make their products more coherent by focusing more on the content and reducing nonmathematical elements. Thus, Sadlier's textbooks focus on priority content for a 140-day school year.

Additionally, research shows that the organization of textbook materials plays an important role in student learning. The objective for a lesson should be clearly spelled out for students at the beginning of a lesson in order to activate students' prior knowledge. Where there are multiple facets to a lesson, an outline or list of the multiple parts of the lesson is even more helpful to the student and the teacher, insofar as either provides an in-depth inventory of the concepts and skills to be addressed. For these reasons, Sadlier's textbooks make lesson objectives clear to the student and the teacher before lessons are taught.

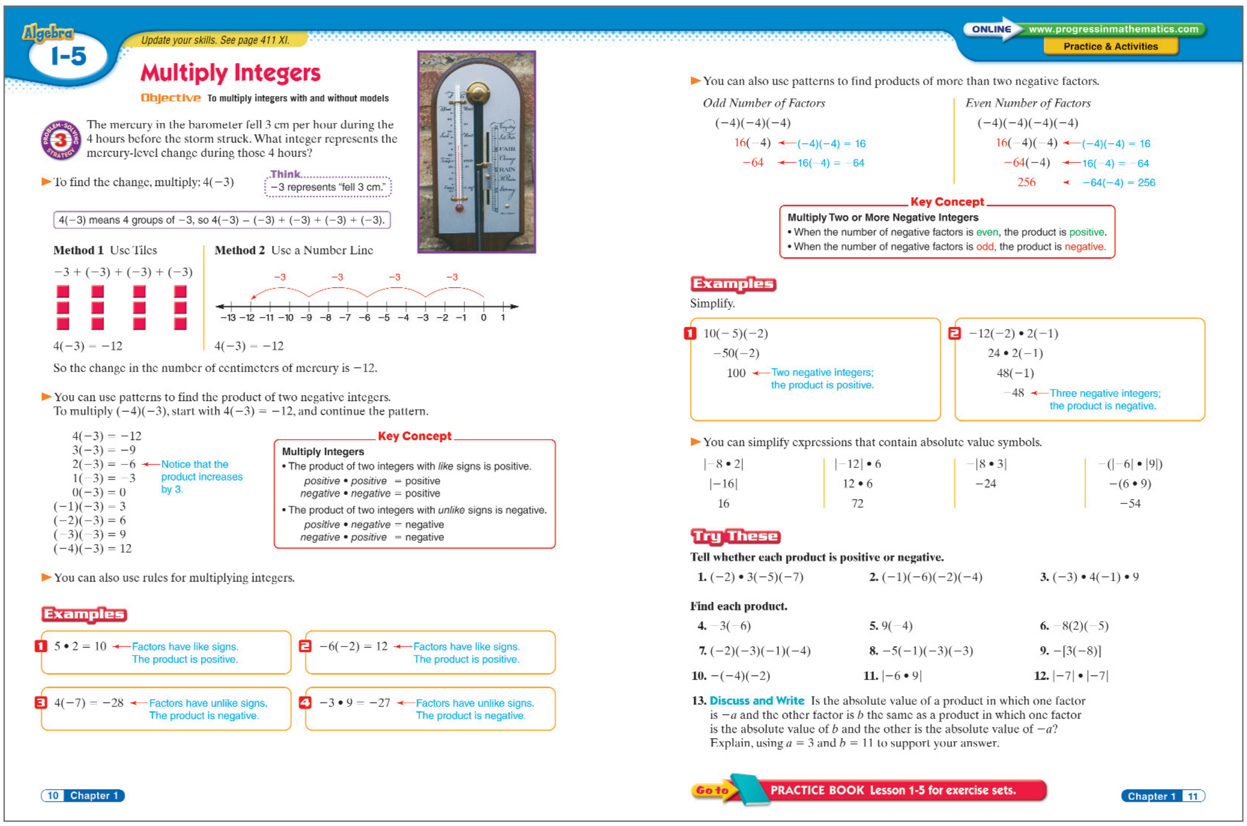
Textbooks need to include carefully chosen, worked-out examples that students can refer to as an alternative to teacher-directed instruction. Also, it is important that these worked-out examples are available to students when they are solving problems, either at school or at home.

The quantity of new conceptual material in a lesson and the evenness of flow from one activity to another within the lesson also contribute to a successful textbook. When each lesson spans a realistic time-on-task for the conceptual load, teachers are more likely to guide students to a new understanding of the material. Also, when transitions from one part of a lesson to another move smoothly, students are less likely to be disengaged from meaningful work.

What the research says. . .

National Mathematics Advisory Panel:

"All parties involved in the publication and adoption of textbooks should strive for more compact and more coherent mathematics texts for use by students in grades K–8 and beyond. . ." (National Mathematics Advisory Panel, 2008, pp. 55). "Textbook publishers should publish editions that include a clear emphasis on the material that states and districts agree to teach in specific grades. . ." (ibid., p. xxiv).



Algebra 1-5

Update your skills. See page 411 XI.

Multiply Integers

Objective To multiply integers with and without models

Problem-Solving Strategy 3

The mercury in the barometer fell 3 cm per hour during the 4 hours before the storm struck. What integer represents the mercury-level change during those 4 hours?

► To find the change, multiply: $4(-3)$

Think: -3 represents "fell 3 cm."

$4(-3)$ means 4 groups of -3, so $4(-3) = (-3) + (-3) + (-3) + (-3)$.

Method 1 Use Tiles

$-3 + (-3) + (-3) + (-3)$

$4(-3) = -12$

Method 2 Use a Number Line

-3 -3 -3 -3

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

$4(-3) = -12$

So the change in the number of centimeters of mercury is -12.

► You can use patterns to find the product of two negative integers. To multiply $(-4)(-3)$, start with $4(-3) = -12$, and continue the pattern.

$4(-3) = -12$
$3(-3) = -9$
$2(-3) = -6$ ← Notice that the product increases by 3.
$1(-3) = -3$
$0(-3) = 0$
$(-1)(-3) = 3$
$(-2)(-3) = 6$
$(-3)(-3) = 9$
$(-4)(-3) = 12$

Key Concept

Multiply Integers

- The product of two integers with *like* signs is positive.
  positive • *positive* = positive
  negative • *negative* = positive
- The product of two integers with *unlike* signs is negative.
  positive • *negative* = negative
  negative • *positive* = negative

► You can also use rules for multiplying integers.

Examples

1. $5 \cdot 2 = 10$ ← Factors have like signs. The product is positive.
2. $-6(-2) = 12$ ← Factors have like signs. The product is positive.
3. $4(-7) = -28$ ← Factors have unlike signs. The product is negative.
4. $-3 \cdot 9 = -27$ ← Factors have unlike signs. The product is negative.

10 Chapter 1

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► You can also use patterns to find products of more than two negative factors.

Odd Number of Factors

$(-4)(-4)(-4)$
$16(-4)$ ← $(-4)(-4) = 16$
-64 ← $16(-4) = -64$

Even Number of Factors

$(-4)(-4)(-4)(-4)$
$16(-4)(-4)$ ← $(-4)(-4) = 16$
$-64(-4)$ ← $16(-4) = -64$
256 ← $-64(-4) = 256$

Key Concept

Multiply Two or More Negative Integers

- When the number of negative factors is even, the product is positive.
- When the number of negative factors is odd, the product is negative.

Examples

Simplify.

1. $10(-5)(-2)$
   $-50(-2)$
   100 ← Two negative integers; the product is positive.
2. $-12(-2) \cdot 2(-1)$
   $24 \cdot 2(-1)$
   $48(-1)$
   -48 ← Three negative integers; the product is negative.

► You can simplify expressions that contain absolute value symbols.

$\lvert -8 \cdot 2\rvert$	$\lvert -12\rvert \cdot 6$	$-\lvert 8 \cdot 3\rvert$	$-(\lvert -6\rvert \cdot \lvert 9\rvert)$
$\lvert -16\rvert$	$12 \cdot 6$	-24	$-(6 \cdot 9)$
16	72		-54

Try These

Tell whether each product is positive or negative.

1. $(-2) \cdot 3(-5)(-7)$
2. $(-1)(-6)(-2)(-4)$
3. $(-3) \cdot 4(-1) \cdot 9$

Find each product.

4. $-3(-6)$
5. $9(-4)$
6. $-8(2)(-5)$
7. $(-2)(-3)(-1)(-4)$
8. $-5(-1)(-3)(-3)$
9. $-[3(-8)]$
10. $-(-4)(-2)$
11. $\lvert -6 \cdot 9\rvert$
12. $\lvert -7\rvert \cdot \lvert -7\rvert$
13. **Discuss and Write** Is the absolute value of a product in which one factor is $-a$ and the other factor is b the same as a product in which one factor is the absolute value of b and the other is the absolute value of $-a$? Explain, using $a = 3$ and $b = 11$ to support your answer.

Go to PRACTICE BOOK Lesson 1-5 for exercise sets.

Chapter 1 11

Grade 7 SourceBook, pages 10–11

"Other potentially useful ways of decreasing length and increasing coherence are (1) reducing the number of photographs that are not essential to the mathematical content; (2) placing content aimed at providing extended review, enrichment activities, or motivation in supplements rather than in the main textbook; and (3) reducing applications in which the primary content is posed by the social studies or science content" (ibid., p. 56).

Individual Studies:

- A study of the use of advance organizers found them to be useful in the learning of verbal learning (Ausubel, 1960).
- A study found that the amount of time spent on instruction correlated positively to academic achievement, whereas the amount of time spent on classroom organization correlated negatively with academic achievement (Brophy, 1988).
- A study found that effective teachers avoided problems with classroom management by keeping students involved in meaningful work throughout a class period. One method that effective teachers used for keeping students engaged was making smooth transitions from one activity to the next within a lesson (Kounin, 1970).

The Critical Foundations for Algebra

Sadlier's Middle School Mathematics program provides:

- pacing that identifies the foundational concepts for algebra as the priority curriculum;
- the modeling of fractions—including number lines—that helps students see and think of fractions as numbers;
- a grade 7 program in which one half of the chapters focus on rational number concepts and procedural knowledge;
- intensive focus on proportional reasoning.

Why?

Since the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* first appeared in 1989, states and school districts have attempted to re-shape their mathematics curricula in order to place less focus on arithmetic computation and more emphasis on topics from the strands of Measurement, Geometry, Algebra, and Data Analysis and Probability. However, the different interpretations given to this curriculum model by various states tended to result in curricula that included many grade-level learning goals at each grade. Teachers often struggled with curricula that were unfocused and repetitive. Once the No Child Left Behind

Act of 2001 passed, high-stakes testing was tied to the overly-broad curricula for the first time, and the subsequent test results made it clear that these curricula were unmanageable.

In 2006, NCTM published the *Curriculum Focal Points* (CFP), a clarification of curriculum goals for Grades K–8 that were based on a new objective: preparing students for Algebra 1 by the end of Grade 8. The CFP embodied a narrowed curriculum of topics that are foundational for Algebra 1, and the NCTM identified this curriculum as the principal curriculum focus at each grade level, K–8. One year later, the NMAP reinforced this perspective in its final report, urging that all parties involved in curriculum planning narrow and prioritize the topics taught at each grade level, K–8. The NMAP identifies the Critical Foundations of Algebra as proficiency with whole number concepts and skills, rational number concepts and skills, and certain areas of geometry and measurement. Among those three foundational areas of concern, rational numbers are identified as the area in which proficiency is the most severely underdeveloped. The important rational number concepts include proportional reasoning, including significant experience with similar triangles.

What the research says. . .

National Mathematics Advisory Panel:

"Proficiency with whole numbers, fractions, and certain aspects of geometry and measurement [is the foundation] for algebra" (National Mathematics Advisory Panel, 2008, p. 19). "Of these, knowledge of fractions is the most important foundational skill not developed among American students" (ibid., p. 18). "As with learning whole numbers, a conceptual understanding of fractions and decimals and the operational procedures for using them are mutually reinforcing. One key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a number line. . ." (ibid., p. 28).

Benchmarks for the Critical Foundations

Sadlier's Middle School Mathematics program provides:

- a scope and sequence that is designed for mastery of the critical foundations in the school year recommended by the NMAP.

Benchmarks for the Critical Foundations[1] (middle school topics indicated in red)

Benchmark	NMAP Expects Mastery	Sadlier Designed for Mastery
Fluency with Whole Numbers		
Students should be proficient in the addition and subtraction of whole numbers.	Grade 3	Grade 3
Students should be proficient in the multiplication and division of whole numbers.	Grade 5	Grade 5
Fluency with Fractions		
Students should be able to identify and represent fractions and decimals, compare them on a number line or with other common representations of fractions and decimals.	Grade 4	Grade 4
Students should be proficient in comparing fractions and decimals and common percent, as well as in the addition and subtraction of fractions and decimals.	Grade 5	Grade 5
Students should be proficient in the multiplication and division of fractions and decimals.	Grade 6	Grade 6
Students should be proficient in all operations involving positive and negative integers.	Grade 6	Grade 6
Students should be proficient in all operations involving positive and negative fractions.	Grade 7	Grade 7
Students should be able to solve problems involving percent and ratio, and they should be able to extend this work to proportionality.	Grade 7	Grade 7
Geometry and Measurement		
Students should be able to solve problems involving the perimeter and area of triangles, and all quadrilaterals having at least one pair of parallel sides.	Grade 5	Grade 6
Students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and they should also be able to analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.	Grade 6	Grade 6
Students should be familiar with the relationship between similar triangles and the concept of the slope of a line.	Grade 7	Grade 7

[1]Derived from National Mathematics Advisory Panel, 2008, p. 20.

Why?

The Critical Foundations of Algebra identified by the NMAP also include benchmarks as to when this foundational material should be taught.

What the research says. . .

National Mathematics Advisory Panel:

"The Benchmarks for the Critical Foundations [in the Table on page 10]. . . should be used to guide classroom curricula, mathematics instruction, and state assessments" (National Mathematics Advisory Panel, 2008, p. 20).

The Major Topics of Algebra

Sadlier's Middle School Mathematics program provides:

- a scope and sequence that is designed for mastery of the Algebra 1 topics recommended by the NMAP;
- additional instruction extending beyond the basic Algebra 1 course, including content that is a normal expectation in Algebra 1 courses of high-achieving countries;
- additional instruction for many topics that are appropriate in honors-level Algebra 1 courses.

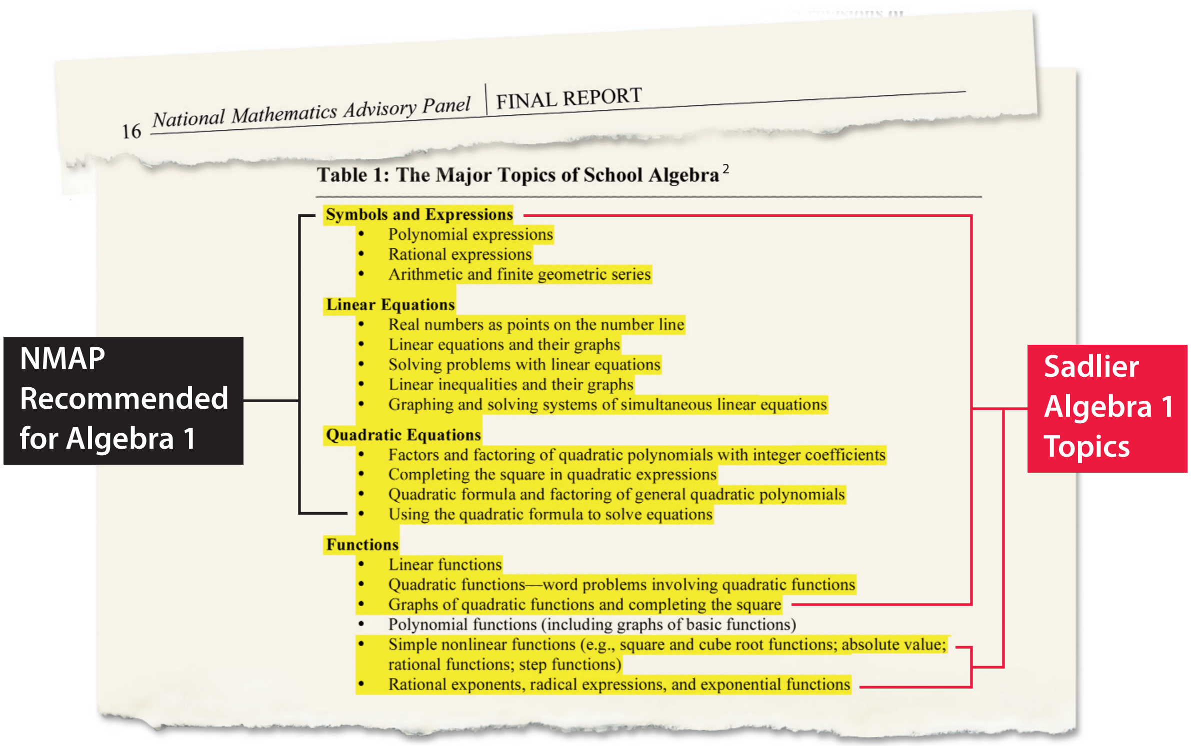

16 *National Mathematics Advisory Panel* | FINAL REPORT

Table 1: The Major Topics of School Algebra[2]

Symbols and Expressions
- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations
- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations
- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions
- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions

[2]National Mathematics Advisory Panel, 2008, p. 16.

Why?

Some of what is taught in Algebra 1 and much of what is taught in Algebra 2 vary greatly across the United States. The NMAP studied which topics are included in different state curriculum frameworks, as well as in the curricula of high-performing countries. The Panel recommended that a core of topics, described as "The Major Topics of Algebra," be the focus of school algebra, or Algebra 1 and Algebra 2. In addition, the Panel observed that the high-performing countries typically include a full discussion of quadratic equations, the derivation of the Quadratic Formula, and the factoring of quadratic equations in their Algebra 1 course.

What the research says. . .

National Mathematics Advisory Panel:

"The Major Topics of School Algebra" on page 11 "should be the focus for school algebra standards in curriculum frameworks, algebra courses, textbooks for algebra, and. . .end-of-course assessments" (National Mathematics Advisory Panel, 2008, p. xvii). "What is usually called Algebra 1 would, in most courses, cover the content in Symbols and Expressions, . . .Linear Equations, and the first two topics in Quadratic Equations. The typical Algebra 2 course would cover the other topics. . ." (National Mathematics Advisory Panel *Reports of the Task Groups and Subcommittees*, 2008, p. 3–4).

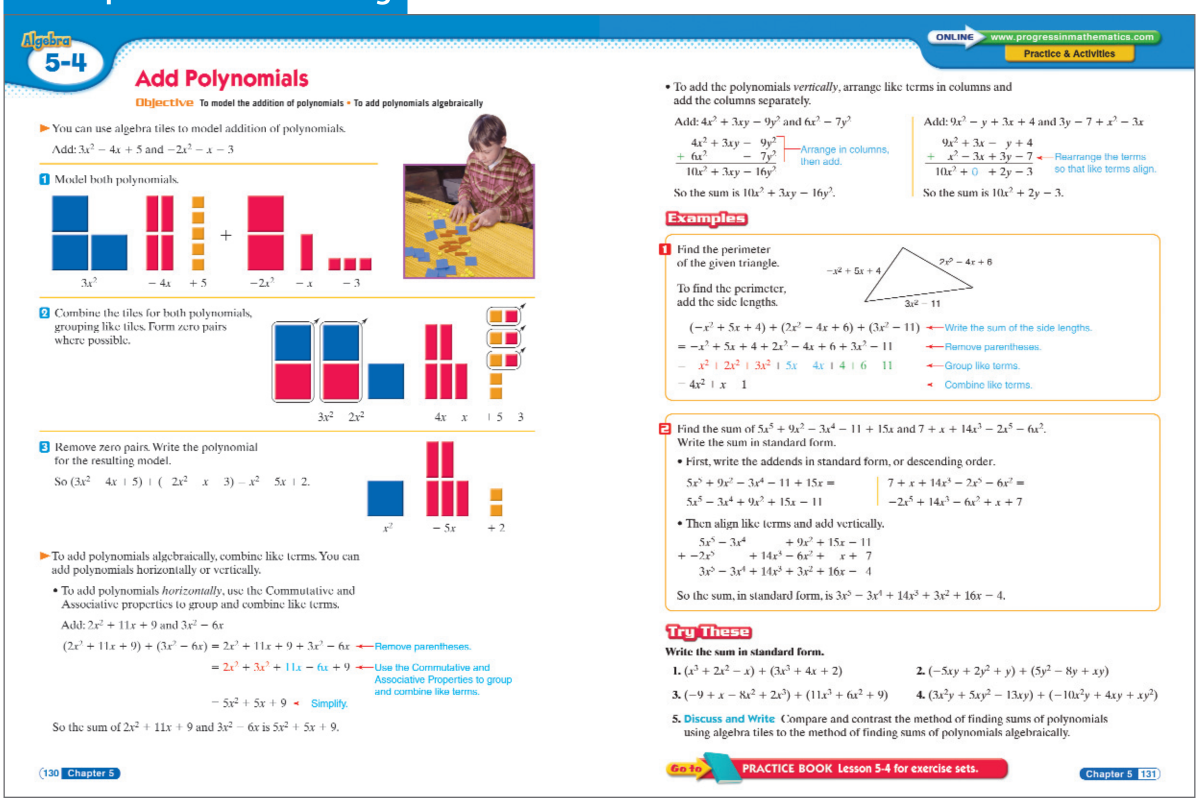
2. Programmatic Features

A Balanced Approach to Achieving Mathematical Proficiency

Sadlier's Middle School Mathematics program provides:

- Instruction that stresses key concepts with appropriate perceptual models that lead to **conceptual understanding**;
- Skill practice that develops **skill fluency** by emphasizing accurate execution of algorithms;
- Regular opportunities to develop **problem-solving competence** through lessons that foster strategic thinking and give students explicit practice in choosing effective problem-solving strategies.

Conceptual Understanding



Algebra 5-4

Add Polynomials

Objective To model the addition of polynomials • To add polynomials algebraically

You can use algebra tiles to model addition of polynomials.

Add: $3x^2 - 4x + 5$ and $-2x^2 - x - 3$

1 Model both polynomials.

$3x^2 \quad -4x \quad +5 \quad + \quad -2x^2 \quad -x \quad -3$

2 Combine the tiles for both polynomials, grouping like tiles. Form zero pairs where possible.

$3x^2 \quad 2x^2 \quad 4x \quad x \quad +5 \quad 3$

3 Remove zero pairs. Write the polynomial for the resulting model.

So $(3x^2 - 4x + 5) + (-2x^2 - x - 3) = x^2 - 5x + 2$.

$x^2 \quad -5x \quad +2$

To add polynomials algebraically, combine like terms. You can add polynomials horizontally or vertically.

- To add polynomials *horizontally*, use the Commutative and Associative properties to group and combine like terms.

Add: $2x^2 + 11x + 9$ and $3x^2 - 6x$

$(2x^2 + 11x + 9) + (3x^2 - 6x) = 2x^2 + 11x + 9 + 3x^2 - 6x$ ← Remove parentheses.

$= 2x^2 + 3x^2 + 11x - 6x + 9$ ← Use the Commutative and Associative Properties to group and combine like terms.

$= 5x^2 + 5x + 9$ ← Simplify.

So the sum of $2x^2 + 11x + 9$ and $3x^2 - 6x$ is $5x^2 + 5x + 9$.

130 Chapter 5

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- To add the polynomials *vertically*, arrange like terms in columns and add the columns separately.

Add: $4x^2 + 3xy - 9y^2$ and $6x^2 - 7y^2$

$$\begin{array}{r} 4x^2 + 3xy - 9y^2 \\ +\ 6x^2 \qquad\quad - 7y^2 \\ \hline 10x^2 + 3xy - 16y^2 \end{array}$$

Arrange in columns, then add.

So the sum is $10x^2 + 3xy - 16y^2$.

Add: $9x^2 - y + 3x + 4$ and $3y - 7 + x^2 - 3x$

$$\begin{array}{r} 9x^2 + 3x - y + 4 \\ +\ x^2 - 3x + 3y - 7 \\ \hline 10x^2 + 0 + 2y - 3 \end{array}$$

← Rearrange the terms so that like terms align.

So the sum is $10x^2 + 2y - 3$.

Examples

1 Find the perimeter of the given triangle.

$-x^2 + 5x + 4 \quad 2x^2 - 4x + 6 \quad 3x^2 - 11$

To find the perimeter, add the side lengths.

$(-x^2 + 5x + 4) + (2x^2 - 4x + 6) + (3x^2 - 11)$ ← Write the sum of the side lengths.

$= -x^2 + 5x + 4 + 2x^2 - 4x + 6 + 3x^2 - 11$ ← Remove parentheses.

$= -x^2 + 2x^2 + 3x^2 + 5x - 4x + 4 + 6 - 11$ ← Group like terms.

$= 4x^2 + x - 1$ ← Combine like terms.

2 Find the sum of $5x^5 + 9x^2 - 3x^4 - 11 + 15x$ and $7 + x + 14x^3 - 2x^5 - 6x^2$. Write the sum in standard form.

- First, write the addends in standard form, or descending order.

$5x^5 + 9x^2 - 3x^4 - 11 + 15x = 5x^5 - 3x^4 + 9x^2 + 15x - 11$

$7 + x + 14x^3 - 2x^5 - 6x^2 = -2x^5 + 14x^3 - 6x^2 + x + 7$

- Then align like terms and add vertically.

$$\begin{array}{r} 5x^5 - 3x^4 \qquad\qquad + 9x^2 + 15x - 11 \\ +\ -2x^5 \qquad + 14x^3 - 6x^2 + x + 7 \\ \hline 3x^5 - 3x^4 + 14x^3 + 3x^2 + 16x - 4 \end{array}$$

So the sum, in standard form, is $3x^5 - 3x^4 + 14x^3 + 3x^2 + 16x - 4$.

Try These

Write the sum in standard form.

1. $(x^3 + 2x^2 - x) + (3x^3 + 4x + 2)$

2. $(-5xy + 2y^2 + y) + (5y^2 - 8y + xy)$

3. $(-9 + x - 8x^2 + 2x^3) + (11x^3 + 6x^2 + 9)$

4. $(3x^2y + 5xy^2 - 13xy) + (-10x^2y + 4xy + xy^2)$

5. Discuss and Write Compare and contrast the method of finding sums of polynomials using algebra tiles to the method of finding sums of polynomials algebraically.

Go to PRACTICE BOOK Lesson 5-4 for exercise sets.

Chapter 5 131

Grade 8 SourceBook, pages 130–131

Skill Fluency

5-4 Add Polynomials

Name ______________________ Date

To add polynomials, combine like terms. You can add polynomials horizontally or vertically.

Add: $(4x^2 + 7x + 9) + (-3x^2 - 4)$

Add polynomials horizontally.

$(4x^2 + 7x + 9) + (\ \ 3x^2 \quad 4)$
$= 4x^2 + (-3x^2) + 7x + 9 + (-4)$ ← Use the Commutative and the Associative Properties to group like terms.
$= x^2 + 7x + 5$ ← Simplify.

So the sum is $x^2 + 7x + 5$.

Add: $(2x^2 + 4xy + 7y^2) + (5x^2 - xy - 8y^2)$

Add polynomials vertically.

$$\begin{array}{r} 2x^2 + 4xy + 7y^2 \\ +\ 5x^2 - \ \ xy - 8y^2 \\ \hline 7x^2 + 3xy - \ \ y^2 \end{array}$$
← Arrange like terms in columns. Then add.

So the sum is $7x^2 + 3xy - y^2$.

Add horizontally or vertically. Write the sum in standard form. You can use algebra tiles to help combine like terms.

1. $(2x^2 - 7x - 3) + (4x^2 - 3x + 6)$
$2x^2 + 4x^2 - 7x - 3x - 3 + 6$
$6x^2 - 10x + 3$

2. $(4x^2 - 7x + 3) + (2x^2 + 2x - 7)$

3. $(3x^2 + 4x \quad 3) + (5x^2 \quad 6x + 3)$

4. $(2x^2 + 6x + 4) + (x^2 \quad 7x + 4)$

5. $(6n^2 + 4n - 3) + (-9n^2 - 4n + 7)$

6. $(2x^2 - 2x - 3) + (3x^2 - 2x + 3)$

7. $(42x^2 - 36x + 64) + (28x^2 + 11x)$

8. $(x^2 - 23x + 14) + (6x^2 - 8)$

9. $(7x + 2) + (4x^2 + 6x - 4)$

10. $(3x^2 + 2x - 7x) + (6x^2 + 4)$

11. $(\ \ 4t^2 \quad 8t + 96) + (30t^2 \quad 102)$

12. $(61x^2 \quad 128) + (44x + 17)$

13. $(45x^3 - 22 + 19x) + (-24x + 17x^3 + 19)$

14. $(5a - 4a^2 + 7) + (12a^2 - 9 - 6a)$

Use with SOURCEBOOK Lesson 5-4, pages 130–131.

Chapter 5 145

For More Practice Go To:
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Add horizontally or vertically. Write the sum in standard form. You can use algebra tiles to help combine like terms.

15. $(3x^2 + 4xy \quad y^2) + (3x^2 \quad 3xy + 9y^2)$
$3x^2 + 3x^2 + 4xy - 3xy - y^2 + 9y^2$
$6x^2 + xy + 8y^2$

16. $(4x^2 + 12xy \quad 3y^2) + (9x^2 \quad 36xy + 7y^2)$

17. $(23w^2x^2 + 3wx + 7) + (-w^2x^2 - 4wx - 8)$

18. $(2yz + 2yz^2 + 3y^2z) + (6y^2z + 2yz^2 - yz)$

19. $(x^2 + 5xy - 14y^2) + (4x^2 - 19y^2)$

20. $(-3c^2 + 8d^2) + (10c^2 + 14cd - 20d^2)$

21. $(23s^2t - 18st^2) + (16s^2t - 13st^2)$

22. $(16ab + a^2b + 20ab^4) + (12ab - 4a^2b - 20ab^4)$

23. $(6x^2y + 4xy) + (3xy^2 + 2x^2y \quad 5xy)$

24. $(2ac + 7a^2c) + (4ac \quad 4a^2c \quad ac^2)$

25. $(\ \ 4a^2b^2 \quad 2ab^2 + 3ab) + (7a^2b^2 \quad 8ab)$

26. $(\ \ 12m^2 \quad 4mn + 4n^2) + (\ \ 8n^2 \quad 2mn)$

27. $(-cd^2 + 4cd - 8c^2d) + (5c^2d - 3cd)$

28. $(4xy^2 - 3x^2y + 4xy) + (3x^2y - 5xy)$

29. $(3a^2c - 5ac^2) + (-2a^2c - 6ac + 5ac^2)$

30. $(7x^2y - 4xy) + (-3xy^2 - 4xy)$

Problem Solving

Write a polynomial in simplified form to represent each situation.

31. The dimensions of a picture are $(p + 7)$ and $(p - 3)$. Kaleigh wants to buy a fabric border for the picture. What length of the fabric border should she buy?

32. If $3a - b$ represents the width of a rectangle and $4a$ represents the length, what is the perimeter of the rectangle?

TEST PREPARATION

33. Evaluate $3c^2d - 2cd + 4d^2$, when $c = -\frac{1}{2}$ and $d = 4$.

A. 81 **B.** 90 **C.** 64 **D.** 71

146 Chapter 5

Grade 8 Practice Book, pages 145–146

Problem-solving Competence

12-10 Problem Solving: Review of Strategies

Read Plan Solve Check

Objective To solve problems by using a variety of strategies

Problem: There are a total of 20 rectangles and triangles on a page of a child's coloring book. Half of the rectangles are squares. In all, the shapes have 74 sides. How many triangles and how many squares are on this page?

Read to understand what is being asked.
List the facts and restate the question.

Facts: There are triangles, squares, and nonsquare rectangles on a page of a coloring book.
There are the same number of squares as nonsquare rectangles.
In all, there are 20 shapes and 74 sides.

Question: How many of the 20 shapes are triangles and how many are squares?

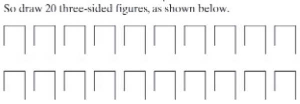
Select a strategy.
You can try using the strategy *Make a Drawing*. Or, you can attempt to *Guess and Test*.

Problem-Solving Strategies
1. Make a Drawing
2. Solve a Simpler Problem
3. Reason Logically
4. Consider Extreme Cases
5. Work Backward
6. Find a Pattern
7. Account for All Possibilities
8. Adopt a Different Point of View
9. Guess and Test
10. Organize Data

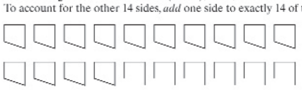
Apply the strategy.

Method 1: Make a Drawing

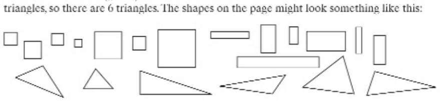
You know that each of the 20 shapes has *at least* three sides. So draw 20 three-sided figures, as shown below.



The drawing accounts for 60 sides ($3 \bullet 20$), or 60, out of the total of 74 sides. To account for the other 14 sides, *add* one side to exactly 14 of the figures.



The 14 four-sided figures represent 14 rectangles, in which half of these rectangles are squares, so there are $\left(\frac{1}{2} \bullet 14\right)$, or 7, squares. The 6 three-sided figures represent the triangles, so there are 6 triangles. The shapes on the page might look something like this:



324 Chapter 12

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Practice & Activities

Method 2: Guess and Test

You can use the *Guess and Test* strategy. Start by guessing the number of squares. From that guess, you can figure out the number of nonsquare rectangles and the number of triangles. Then you can figure out the number of sides. If it is not 74, adjust your guess and try again.

- Start by guessing that there are 2 squares.

Number of squares: 2
Number of nonsquare rectangles: 2 ← Half the rectangles are not squares.
Number triangles: 16 ← There are 20 shapes in all.
Total number of sides: 64 ← Each square and nonsquare rectangle has 4 sides; each triangle has 3 sides: $2(4) + 2(4) + 16(3) = 64$

The number of sides is too small.

- Increase the guess for the number of squares. Try 8.

Number of squares: 8
Number of nonsquare rectangles: 8 ← Half the rectangles are not squares.
Number triangles: 4 ← There are 20 shapes in all.
Total number of sides: 76 ← $8(4) + 8(4) + 4(3) = 76$

There are too many sides.

- Continue guessing and testing until the total number of sides is 74. You can make a table, like the one below, to keep track of your work.

Number of Squares	Number of Nonsquare Rectangles	Number of Triangles	Number of Sides	
2	2	16	64	too few sides
8	8	4	76	too many sides
5	5	10	70	too few sides
6	6	8	72	close
7	7	6	74	Correct!

So there are 7 squares and 6 triangles on the page.

Check to make sure your answer makes sense.

- Are there 20 shapes in all?

7 squares + 7 nonsquare rectangles + 6 triangles = 20 shapes ✓

- Are there 74 sides in all?

$(7 \bullet 4)$ sides for the squares → 28 sides
$(7 \bullet 4)$ sides for the nonsquare rectangles → 28 sides
$+ (6 \bullet 3)$ sides for the triangles → + 18 sides
74 sides ✓

The answer checks.

Go to PRACTICE BOOK Lesson 12-10 for exercise sets.

Chapter 12 325

Algebra 1 SourceBook, pages 324–325

Why?

Debates have raged for decades over which is more valuable for achieving mathematical proficiency: conceptual development or computational proficiency. The NMAP describes this debate as "misguided" and recommends that educators view **conceptual understanding**, **skill fluency**, and **problem-solving competence** as mutually supportive. This conclusion also reinforces the findings of the Mathematics Learning Study Committee of the National Research Council's (NRC, 2001) Center for Education. Its review of mathematics learning from pre-kindergarten through grade 8 identified several "strands of mathematical proficiency. . .[that are] interwoven and interdependent":

1. *Conceptual understanding*—comprehension of mathematical concepts, operations, and relations;
2. *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
3. *Strategic competence*—ability to formulate, represent, and solve mathematical problems;
4. *Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification;
5. *Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

What the research says. . .

National Mathematics Advisory Panel:

"Conceptual understanding, computational and procedural fluency, and problem-solving skills are equally important and mutually reinforce one another" (National Mathematics Advisory Panel, 2008, p. 19).

Individual Studies:

- A review of the relationship between conceptual and procedural knowledge found that conceptual knowledge is related to conceptual skill.

  "Children's understanding of mathematical concepts is positively correlated with their ability to execute procedures. In some tasks, conceptual understanding precedes procedural competence; in other tasks, the order is reversed" (Rittle-Johnson & Siegler, 1998, p. 109).

- A study of the relationship between conceptual knowledge and procedural knowledge noted the importance of inculcating both types of knowledge in the classroom.

". . .Improved procedural knowledge can lead to improved conceptual knowledge, as well as the reverse" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 360).

Explicit and Systematic Instruction

Sadlier's Middle School Mathematics program provides:

- an instructional approach that regularly includes explicit instruction, teacher modeling, guided practice, and application;
- clear models for each instructional concept, using a thorough, carefully chosen set of different examples;
- step-by-step support for teaching the examples, including suggested questions and modeling ideas;
- extensive practice.

Why?

Explicit and systematic instruction refers to the direct teaching of mathematical concepts and skills in a clearly-defined sequence designed to grow over the course of the school year and from grade to grade. A logically developed progression of concepts and skills ensures that students are able to learn more and more complex material as they build on the foundation of previously learned concepts and skills. Direct teaching of a logical sequence of concepts and skills extends students' knowledge systematically in mathematically effective ways. However, within a systematic instruction plan, a combination of "teacher-directed" approaches, such as direct instruction, modeling, guided practice, and application, can be combined with "guided discovery" to produce a beneficial effect on learning. In general, exclusive reliance on either teacher-directed instructional approaches or student-centered approaches has not been shown to be supported by research, according to the NMAP. The Panel recommends that neither be used as the sole approach to instruction. However, the Panel did find that explicit instruction has a demonstrated value with certain student groups and in certain circumstances. Two such groups are advanced students and at-risk students.

With respect to Algebra and Pre-Algebra, students gain in their ability to translate words into algebraic terms by studying explicit written model solutions and then practicing the translations.

What the research says. . .

National Mathematics Advisory Panel:

"Explicit instruction for students who struggle with math is effective in increasing student learning. Teachers should understand how to provide clear models for solving a problem type using an array of examples, [and they should] offer opportunities for extensive practice, encourage students to 'think aloud,' and give specific feedback" (National Mathematics Advisory Panel, 2008, p. xxiii).

Research Review:

- In a review of high-quality studies in mathematics education, the National Center to Improve the Tools of Educators found support using both direct teaching and guided discovery in lessons. In studies of effective strategies, it also found support for selecting and sequencing instructional examples according to principles of concept acquisition. It found no advantages for strictly discovery instruction (Dixon, Carnine, Lee, Wallin, & Chard, 1998).
- In a study done with high-school algebra students, the study found that students who studied several examples of writing equations from written verbal descriptions of mathematical relationships showed greater success at writing equations than students who did not study worked-out examples (Carroll, 1994).

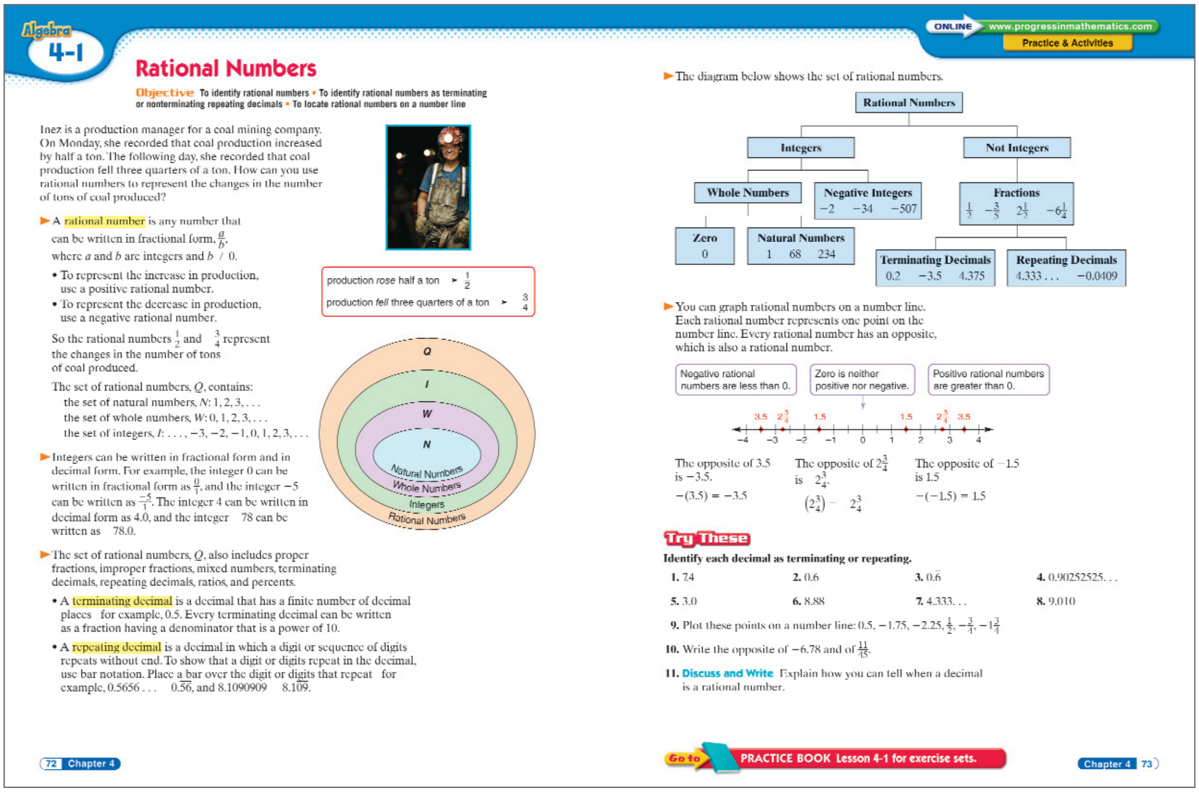
Support for Visual Learning

Sadlier's Middle School Mathematics program provides support for the development of the core concepts through different representations of them:

- multiple representations of concepts through different models;
- extensive use of concept maps, diagrams, and flow charts;
- explicit instruction in how to understand and use graphic representations;
- an extensive focus on relating different representations of linearity: tables, graphs, and equations;
- an extensive focus on the visual representation of data;
- extensive use of technology in the form of online and handheld resources to enhance visual learning.

Why?

Several generations ago, the material in textbooks was mostly a combination of verbal explanation and visual support in the form of line drawings. Over the last several decades, however, photographs have been increasingly used for visual representations of data, and photographs tend to contain information that is both relevant and irrelevant to the concepts the textbook is trying to illustrate. Representations that contain a great deal of irrelevant information are more difficult to interpret. As a result, students need more help. Research has shown that the ability to visualize does not develop by itself. Because representations by means of photographs increasingly need to be interpreted, both textbooks and teachers play a role in providing extended support as students internalize new representations of concepts. Moreover, although, students remember information better when presented



Algebra 4-1

Rational Numbers

Objective To identify rational numbers • To identify rational numbers as terminating or nonterminating repeating decimals • To locate rational numbers on a number line

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Inez is a production manager for a coal mining company. On Monday, she recorded that coal production increased by half a ton. The following day, she recorded that coal production fell three quarters of a ton. How can you use rational numbers to represent the changes in the number of tons of coal produced?

► A rational number is any number that can be written in fractional form, $\frac{a}{b}$, where a and b are integers and b / 0.

- To represent the increase in production, use a positive rational number.
- To represent the decrease in production, use a negative rational number.

production *rose* half a ton ➤ $\frac{1}{2}$
production *fell* three quarters of a ton ➤ $\frac{3}{4}$

So the rational numbers $\frac{1}{2}$ and $\frac{3}{4}$ represent the changes in the number of tons of coal produced.

The set of rational numbers, Q, contains:
the set of natural numbers, N: 1, 2, 3, . . .
the set of whole numbers, W: 0, 1, 2, 3, . . .
the set of integers, I: . . . , −3, −2, −1, 0, 1, 2, 3, . . .

► Integers can be written in fractional form and in decimal form. For example, the integer 0 can be written in fractional form as $\frac{0}{1}$, and the integer −5 can be written as $\frac{-5}{1}$. The integer 4 can be written in decimal form as 4.0, and the integer 78 can be written as 78.0.

► The set of rational numbers, Q, also includes proper fractions, improper fractions, mixed numbers, terminating decimals, repeating decimals, ratios, and percents.

- A terminating decimal is a decimal that has a finite number of decimal places for example, 0.5. Every terminating decimal can be written as a fraction having a denominator that is a power of 10.
- A repeating decimal is a decimal in which a digit or sequence of digits repeats without end. To show that a digit or digits repeat in the decimal, use bar notation. Place a bar over the digit or digits that repeat for example, 0.5656 . . . $0.\overline{56}$, and 8.1090909 $8.1\overline{09}$.

72 Chapter 4

► The diagram below shows the set of rational numbers.

► You can graph rational numbers on a number line. Each rational number represents one point on the number line. Every rational number has an opposite, which is also a rational number.

The opposite of 3.5 is −3.5.
−(3.5) = −3.5

The opposite of $2\frac{3}{4}$ is $2\frac{3}{4}$.
$(2\frac{3}{4})$ − $2\frac{3}{4}$

The opposite of −1.5 is 1.5
−(−1.5) = 1.5

Try These

Identify each decimal as terminating or repeating.

1. 7.4
2. 0.6
3. $0.\overline{6}$
4. 0.90252525. . .
5. 3.0
6. 8.88
7. 4.333. . .
8. 9.010
9. Plot these points on a number line: 0.5, −1.75, −2.25, $\frac{1}{2}$, $-\frac{3}{4}$, $-1\frac{3}{4}$
10. Write the opposite of −6.78 and of $\frac{11}{15}$.
11. **Discuss and Write** Explain how you can tell when a decimal is a rational number.

Go to PRACTICE BOOK Lesson 4-1 for exercise sets.

Chapter 4 73

Grade 7 SourceBook, pages 72–73

in a graphic representation than they do if it is presented with colorful pictures, students visualize in different ways. Thus, it is important that they be exposed to different representations of important concepts.

In the middle grades, the opportunities for students to utilize representations to recall concepts come in many forms: (1) models for concepts relating to rational numbers of all types—fractions, decimals, and percents; (2) representations that show relationships in data sets; (3) representations that apply proportionality; (4) representations of linearity through tables, equations, and graphs; and (5) representations of hierarchical and procedural relationships.

What the research says

Individual Studies:

- A study of different methods of visualization among students concluded that students need extended support to acquire the ability to visualize (Grinder, 1992).
- A study of different types of supplementary materials compared the effects on students' memory of graphic representations (flow charts with key words) versus colorful pictures. Students remembered more if they were given the graphic representation (Imhof, Echtrernach, Huber, & Knorr, 1996).

Problem Solving

Sadlier's Middle School Mathematics program provides:

- an approach to problem solving that encourages self-monitoring during problem solving. Strategies are learned in a step-by-step approach in which

  (1) each step of the strategy is clearly modeled;

  (2) opportunities are provided for students to see and compare different strategies for solving the same problem;

  (3) opportunities are provided for students to solve problems that draw on a variety of previously learned strategies.
- opportunities to apply strategies are also supported in lessons that are not the explicit focus of problem solving;
- a heavy emphasis on representing mathematical relations.

Why?

Problem solving in a mathematics class involves more than the routine application of a newly learned skill to a word problem. Problem solving may require the use of previously learned concepts or skills, recently learned concepts or skills, and/or the simultaneous application of multiple skills and concepts. In solving a problem, students analyze the information given in the problem, consider the range of strategies they know for solving a problem, decide on the range of strategies that best address the problem as they have analyzed it, or develop new strategies for solving the problem. Students acquire flexibility in using the strategies they know or in developing new strategies through practice—specifically, practice in solving problems in a variety of formats and contexts and at increasing levels of complexity and by utilizing the depth of their content knowledge to expand the range of their strategic thinking. Opportunities for students to see and compare different strategies for solving the same problem help students develop the habits of self-regulation that are crucial to successful problem solving. Graphic and semantic organizers as well as mental imagery can be helpful tools when examining and representing relationships in a problem.

What the research says. . .

Research Review:

- Skill in representing mathematical relations spatially and skill in translating relations from verbal form to equations are important tools in problem solving.
- "Strong mathematical problem solving [ability] appears to be associated with, among others, the ability to spatially represent mathematical relations, the ability to translate word problems into appropriate equations, and an understanding of how and when to use mathematical equations" (Geary, 1995).

Learning a Strategy

1-15 Problem-Solving Strategy: Make a Drawing

ONLINE www.progressinmathematics.com Practice & Activities

Objective To solve problems using the strategy *Make a Drawing*

Problem 1: The garage at a repair shop contains motorcycles, pickup trucks, and cars. There are 14 vehicles in all, and there are two more cars than trucks. If the vehicles have a total of 44 wheels, how many vehicles of each type are there?

Read **Read to understand what is being asked.**
List the facts and restate the question.

Facts: There are a total of 14 motorcycles, pickup trucks, and cars.
There are 2 more cars than trucks.
Together, the vehicles have 44 wheels.

Question: How many vehicles of each type are there?

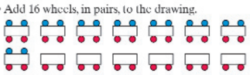
Plan **Select a strategy.**
You could guess the number of vehicles of each type and then count the wheels. However, it is easier and more efficient to use the strategy *Make a Drawing*.

Problem-Solving Strategies
1. Make a Drawing
2. Organize Data
3. Guess and Test
4. Find a Pattern
5. Reason Logically
6. Solve a Simpler Problem
7. Adopt a Different Point of View
8. Work Backward
9. Account for All Possibilities
10. Consider Extreme Cases

Solve **Apply the strategy.**
- Draw 14 rectangles to represent the vehicles.

Each vehicle must have *at least* two wheels.
- Draw two wheels on each vehicle.

There are 28 wheels so far and 44 − 28, or 16 wheels, left to account for.
- Add 16 wheels, in pairs, to the drawing.



The drawing shows that there are 6 two-wheeled vehicles. These must be motorcycles. The drawing shows that there are 8 four-wheeled vehicles. Since there are two more cars than trucks, there must be 5 cars and 3 trucks.

Check **Check to make sure your answer makes sense.**
Are there 14 vehicles in all? Yes, 5 + 3 + 6 = 14 ✓
Are there two more cars than trucks? Yes, 5 − 3 = 2 ✓
Are there 44 wheels in all? Yes, 5(4) + 3(4) + 6(2) = 20 + 12 + 12 = 44 ✓

30 Chapter 1

Problem 2: Archeologists recently unearthed documents about an obscure 12th-century king, Olafmola, who had six children. The king apparently gave one fourth of his land to his eldest daughter, the crown princess. The remaining land was then divided into nine equal parts. His other daughter and four sons each received one part; the rest was kept for the king's court and the royal farm. What portion of the original land did the king reserve for the court and farm?

Read **Read to understand what is being asked.**
List the facts and restate the question.

Facts: $\frac{1}{4}$ of the king's land went to his eldest daughter.
$\frac{5}{9}$ of the land that remained was given to his other children.
The rest was kept for the court and farm.

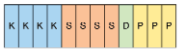
Question: What portion of the king's land was reserved for the court and farm?

Plan **Select a strategy.**
Using the strategy *Make a Drawing* can help you visualize how the land was divided.

Solve **Apply the strategy.**
- Draw a rectangle to represent all of the king's land. Divide the rectangle into four equal parts. Label the drawing to show that one part goes to the crown princess (*P*).

- Divide each of the three parts that remain into three equal parts to create a total of nine equal parts. Assign five of these parts to the remaining daughter (D) and four sons (S). The parts that remain go to the king's court and farm (K).

- Divide the portion set aside for the crown princess into the same three equal parts as the others. The land will then be divided into 12 equal parts. You can see that the portion that goes to the king's court and farm is $\frac{4}{12}$, or $\frac{1}{3}$, of the original land.



Check **Check to make sure your answer makes sense.**
The king divided his land by giving $\frac{1}{4}$ of it to the crown princess. This left $\frac{3}{4}$ of his land. He then kept $\frac{4}{9}$ of this remaining $\frac{3}{4}$ for his court and farm. $\frac{4}{9}$ of $\frac{3}{4}$ is $\frac{12}{36}$, or $\frac{1}{3}$. This agrees with the answer we found by making a drawing.

Go to PRACTICE BOOK Lesson 1-15 for exercise sets.

Chapter 1 31

Grade 8 SourceBook, pages 30–31

Choosing a Strategy

4-12 Problem Solving: Review of Strategies

Read Plan Solve Check

ONLINE www.progressinmathematics.com Practice & Activities

Objective To solve problems by using a variety of strategies

Problem 1: Hose A can fill a "kiddie pool" in 6 minutes. Hose B can fill the same pool in 9 minutes. If the two hoses are used together, how long will it take to fill this pool?

Read to understand what is being asked.
List the facts and restate the question.

Facts: By itself, Hose A can fill a pool in 6 minutes.
By itself, Hose B can fill a pool in 9 minutes.
The hoses will be used together.

Question: How long will it take to fill the pool?

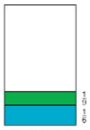
Problem-Solving Strategies
1. Make a Drawing
2. Organize Data
3. Guess and Test
4. Find a Pattern
5. Reason Logically
6. Solve a Simpler Problem
7. Adopt a Different Point of View
8. Work Backward
9. Account for All Possibilities
10. Consider Extreme Cases

Select a strategy.
There are many ways to approach this problem. Here are two possibilities:
- You can *Make a Drawing* to better understand the problem.
- You can *Organize Data* to analyze the situation.

Apply the strategy.

▶ Method 1: Make a Drawing
- Draw a rectangle to represent the pool. After 1 minute, Hose A will have filled $\frac{1}{6}$ of the pool, and Hose B will have filled $\frac{1}{9}$ of the pool. Show this in the drawing. (Your drawing does not have to be accurate. It can just be a sketch.)

  After 1 minute, $\frac{1}{6} + \frac{1}{9}$ of the pool will be full.

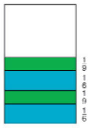


After 1 minute
$(\frac{1}{6} + \frac{1}{9})$ full

- In the second minute, Hose A will have filled another $\frac{1}{6}$ of the pool and Hose B will have filled another $\frac{1}{9}$. Add this to the drawing. After 2 minutes, $2(\frac{1}{6} + \frac{1}{9})$ of the pool will be full.

  Each minute, another $\frac{1}{6} + \frac{1}{9}$ will be filled. So in t minutes, $t(\frac{1}{6} + \frac{1}{9})$ of the pool will be full.

  When the pool is full, $t(\frac{1}{6} + \frac{1}{9})$ will be equal to 1 (because it will represent 1 whole pool).



After 2 minutes
$2(\frac{1}{6} + \frac{1}{9})$ full

- Solve: $t(\frac{1}{6} + \frac{1}{9}) = 1$

$$t\left(\frac{5}{18}\right) = 1 \quad \text{Add: } \frac{1}{6} + \frac{1}{9} = \frac{3}{18} + \frac{2}{18} = \frac{5}{18}$$
$$t\left(\frac{5}{18}\right)\left(\frac{18}{5}\right) = 1\left(\frac{18}{5}\right) \quad \text{Multiply both sides by } \frac{18}{5}.$$
$$t = \frac{18}{5} = 3\tfrac{3}{5} \quad \boxed{\text{solution}}$$

118 Chapter 4

- You can write a different equation if you look at the problem in another way.
  Let t = the number of minutes it takes both hoses to fill the pool.
  $\frac{1}{t}$ = the amount the pool will be filled by both hoses in 1 min

$$\frac{1}{6} + \frac{1}{9} = \frac{1}{t} \quad \longleftarrow \text{In 1 min, both hoses can fill } \frac{1}{6} + \frac{1}{9}, \text{ or } \frac{1}{t}.$$
$$\frac{5}{18} = \frac{1}{t} \quad \longleftarrow \text{Add.}$$
$$5t = 18 \quad \longleftarrow \text{Cross multiply.}$$
$$t = \frac{18}{5} = 3\tfrac{3}{5} \quad \longleftarrow \boxed{\text{solution}}$$

Think
$3\frac{3}{5}$ min = 3 min 36 s

So it will take $3\frac{3}{5}$ min, or 3 min 36 s, for Hose A and Hose B together to fill the pool.

▶ Method 2: Organize Data
You can make a table like the one below to analyze the situation.

Time (minutes)	3	6	9	12	15	18
Pools Filled by Hose A	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
Pools Filled by Hose B	$\frac{1}{3}$	$\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2
Total Pools Filled	$\frac{5}{6}$	$1\frac{2}{3}$	$2\frac{1}{2}$	$3\frac{1}{3}$	$4\frac{1}{6}$	5

- The first row shows the amount of time in 3-minute increments. The increment of 3 is used because 3 is a common factor of 6 and 9.
- The next two rows show the number of pools each hose, working alone, can fill in each amount of time.
- The last row shows the total number of pools filled by the two hoses together in each amount of time.
- The last column shows that the two hoses can fill 5 pools in 18 min.
  Divide $\frac{18}{5}$ to find the time to fill one pool.
  So filling one pool would take $\frac{18}{5}$ min, or 3 min 36 s.

Check to make sure your answer makes sense.
In $\frac{18}{5}$ min, Hose A fills $\frac{\frac{18}{5}}{6}$ of the pool, and Hose B fills $\frac{\frac{18}{5}}{9}$. If $\frac{18}{5}$ min is the correct answer, then $\frac{\frac{18}{5}}{6} + \frac{\frac{18}{5}}{9} = 1$.

$$\frac{\frac{18}{5}}{6} + \frac{\frac{18}{5}}{9} = \frac{18}{5} \bullet \frac{1}{6} + \frac{18}{5} \bullet \frac{1}{9}$$
$$= \frac{\overset{3}{\cancel{18}}}{5} \bullet \frac{1}{\underset{1}{\cancel{6}}} + \frac{\overset{2}{\cancel{18}}}{5} \bullet \frac{1}{\underset{1}{\cancel{9}}}$$
$$= \frac{3}{5} + \frac{2}{5} = 1 ✓$$

Go to PRACTICE BOOK Lesson 4-12 for exercise sets.

Chapter 4 119

Grade 8 SourceBook, pages 118-119

Individual Study:

- A common problem for students, even at the high-school and college level, is their inability to monitor whether the problem-solving strategy they have selected is working.

  "In Schoenfeld's collection of (more than a hundred) videotapes of college and high-school students working unfamiliar problems, roughly 60% of the solution attempts are of the 'read, make a decision quickly, and pursue that direction come hell or high water' variety" (Grouws, 1992, p. 356).

Formative Assessment

Sadlier's Middle School Mathematics program provides:

- regular formative assessments called "Check Your Progress" that appear every 3–6 lessons;
- practice Chapter Tests, Beginning-of-Year Tests, Quarterly Tests, and End-of-Year Tests that reveal concepts and skills in need of additional work;
- cumulative reviews that serve as benchmark assessments.

Why?

The value of formative assessments—measures of student learning that guide and redirect ongoing instruction—has been shown in many studies. Formative assessments allow the teacher to avoid squandering instructional time on concepts that are largely mastered, to place additional emphasis on concepts and skills that students are struggling with, and to individualize the focus of instruction. The NMAP recommends "regular use of formative assessment for students in the elementary grades."

What the research says...

National Mathematics Advisory Panel:

"Teachers' regular use of formative assessments can improve student learning in mathematics" (National Mathematics Advisory Panel, 2008, p. 46).

Individual Study:

- In the King's-Medway-Oxfordshire Formative Assessment Project (KMOFAP) and in a parallel project at Stanford University, students who had received feedback based on formative assessments showed considerable growth on standardized tests (Black, Harrison, Lee, Marshall, & Wiliam, 2003).

3. Product Development Research

Prior to the publication of Sadlier's Middle School Mathematics program, each stage of development of the program was subjected to scrutiny by educators at all levels. Teachers, mathematics coaches, department heads, principals, and supervisors reviewed the program and offered suggestions. Authors provided the architecture of the program and critiqued each stage. Sadlier's Mathematics Advisory Board, an organization comprised of mathematicians, mathematics educators, researchers, and educational specialists, offered guidance throughout the writing of the program.

Stage 1: School Visits and Interviews with Classroom Teachers

School visits and reviews with classroom teachers took place in four states (Florida, New York, New Jersey, and Pennsylvania) and included teachers from both public and nonpublic schools. Many important product concepts came from these initial discussions. For example, it was at this point that teachers expressed their interest in a product configuration built around a separate SourceBook and Practice Book for students.

Stage 2: Focus Groups with Educators

Focus-group sessions with teachers and supervisors were conducted in several states. These sessions offered educators a chance to review prototype materials and prospective Tables of Contents for the program. Utilizing the feedback from these sessions, the program developers created more detailed lesson-by-lesson tables of contents and sample chapters of prototype materials.

Stage 3: Teacher Review of Prototype Materials

Teacher reviewers inspected and offered suggestions on the lesson-by-lesson Tables of Contents for all three levels. They also reviewed sample chapters from each of the three levels of the program.

Stage 4: Systematic Review of Developing Manuscript

The author team, some members of the Mathematics Advisory Board, teachers, and other consultants reviewed stages of the manuscript in a systematic way to ensure that the pedagogy and mathematical integrity of the program were of the highest quality.

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